International Kangaroo Mathematics Contest 2008

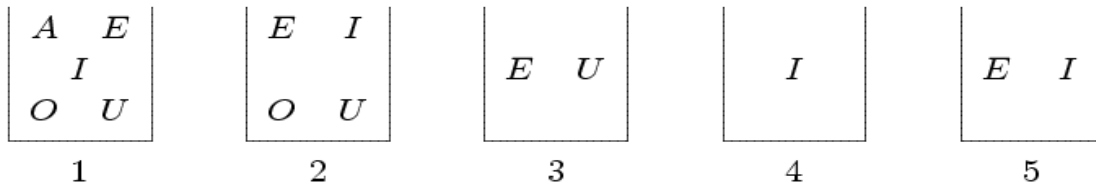*Junior Level: Class (9 & 10)*   *Max Time: 2 Hours*

3-point problems

1)

There are 5 boxes and each box contains some cards labelled A, E, I, O, U as shown. Aslam wants to remove cards from each box in such a way that at the end each box contains only one card, and different boxes contain cards with different letters. What card remains in box 2?

A E I O U	E I O U	E U	I	E I
1	2	3	4	5

A) A   B) E   C) I   D) O   E) U

2)

Amir and Ahmad did compete in running on 200 meters. Ahmad did it under half of the minute, but Amir did it under the hundredth part of one hour. Who and by how many seconds was faster?

A) Ahmad by 36 seconds   B) Amir by 24 seconds
C) Ahmad by 6 seconds   D) Amir by 4 seconds
E) They did it by equal time

3)

To greet the New Year day 2008, Akram put on a T-shirt with ⧖⧗⧗⧖ printed on it, and stood in front of a mirror on his hands, with his feet up. What was seen in the mirror by his friend Zahid, who stood (on this feet) beside Akram?

A)   B)   C)   D)   E)

2008   5008   8002   8005   2005

4)

$a = 2 - (-4)$, $b = (-2)(-3)$, $c = 2 - 8$, $d = 0 - (-6)$ and $e = (-12) \div (-2)$

How many of these results are not equal to 6?
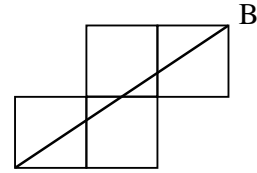
A) 0   B) 1   C) 2   D) 4   E) 5

5)

What is the length of line AB if the side of each of the four squares shown is 1m?

A) 5

B) $\sqrt{13}$

C) $\sqrt{5} + \sqrt{2}$

D) $\sqrt{5}$

E) none of the previous

6)

What smallest number of letters should be removed from the word KANGOUROU so that the remaining letters go in the alphabetical order?

A) 1   B) 2   C) 3   D) 4   E) 5

7)

Each letter is a digit, a digit is only one letter, which digit is K ?

A) 0   B) 1   C) 2   D) 8   E) 9

8)

Tom and Jerry cut two equal rectangles. Tom got two rectangles with the perimeter of 40 cm each, and Jerry got two rectangles with the perimeter of 50 cm each. What were the perimeters of the initial rectangles?
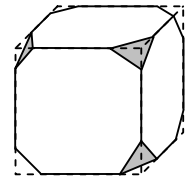
A) 40 cm   B) 50 cm   C) 60 cm   D) 80 cm   E) 100 cm

9)

A cube has all its corners cut off, as shown. How many edges does the resulting shape have?

A) 26   B) 30   C) 36   D) 40
E) Another answer

10)

On my first spelling test, I score one mark out of five. If I now work hard and get full marks on every test, how many more tests should I take for my average to be four out of five correct answers?

A) 2   B) 3   C) 4   D) 5   E) 6

4-point problems

11)

Seven cards lie in a box. Numbers from 1 to 7 are written on these cards (exactly one number on the card). The first sage takes, at random, 3 cards from the box and the second sage takes 2 cards (2 cards are left in the box). Then the first sage tells to the second one: "I know that the sum of the numbers of your cards is even". The sum of card's numbers of the first sage is equal to
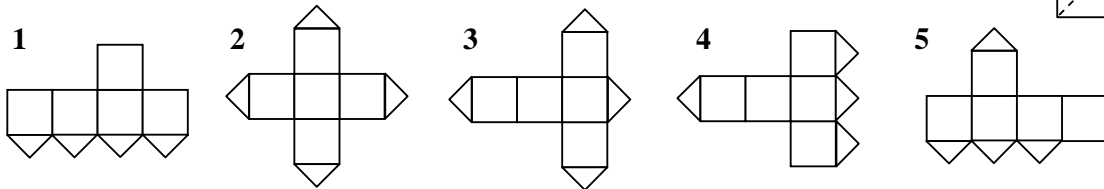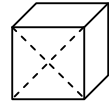
A) 10   B) 12   C) 6   D) 9   E) 15

12)

Bilal has 10 cards, on each exactly one of the numbers 3, 8, 13, 18, 23, 28, 33, 48, 53, 68 is written. What the least number of these cards should Bilal choose so that the sum of the numbers on the chosen cards is equal to 100?

A) 2   B) 3   C) 4   D) 5   E) it is impossible to do

13)

One of the cube faces is cut along its diagonals (see the fig.). Which of the following net is impossible?

1   **2**   **3**   **4**   **5**

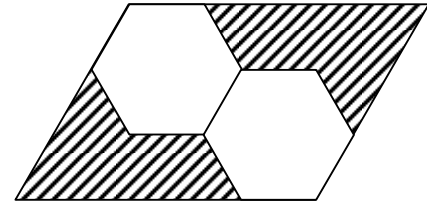A) 1 and 3   B) 1 and 5   C) 3 and 4   D) 3 and 5   E) 2 and 4

14)

The 7 dwarfs was born on the same day, in 7 consecutive years. The 3 youngest of them are 42 years old together. How many years old are the 3 oldest together?

A) 51   B) 54   C) 57   D) 60   E) 63

15)

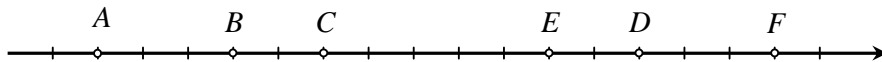In the figure the two regular hexagons are equal to each other. What fraction of the parallelogram's area is shaded?

A) $\dfrac{1}{2}$ B) $\dfrac{1}{3}$ C) $\dfrac{1}{4}$ D) $\dfrac{1}{5}$ E) $\dfrac{1}{6}$

16)

Six integers are marked on the real line (see the fig.). It is known that at least two of them are divided by 3, and at least two of them are divided by 5. Which numbers are divided by 15?

A B C E D F

A) A and F B) B and D C) C and E
D) all six numbers   E) only one of them

17)

How many prime numbers p have the property that $p^4 + 1$ is prime as well?

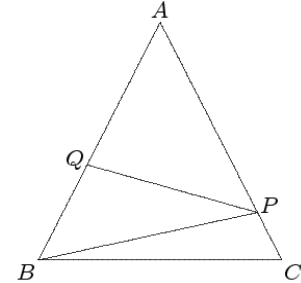A) None   B) 1   C) 2   D) 3   E) Infinitely many

18)

How many digits can be at most erased from the 1000-digit number 20082008…2008, such that the sum of the remained digits is 2008?

  A) 260   B) 510   C) 520   D) 1020   E) 130

19)

The picture shows an isosceles triangle ABC with AB=AC. If PQ is perpendicular to AB, angle BPC is 120° and angle ABP is 50° then what is angle PBC?

  A) 5° B) 10° C) 15°

  D) 20° E) 25^0

20)

How many pairs of real numbers exist such that the sum, the product, and the quotient of these two numbers is equal?

  A) no pair   B) 1 pair   C) 2 pairs   D) 4 pairs   E) 8 pairs

5-point problems

21)

Each digit, starting from the third one, in the decimal representation of a six-digit number is equal to the sum of two previous digits. How many six-digit numbers possess this property?

  A) no one   B) 1   C) 2   D) 4   E) 6

22)

I have a wooden cube, with three red sides and three blue. When cutting this cube into 3 x 3 x 3 = 27 equal small cubes, how many of these have at least 2 sides one of which is red the other one blue?

  A) 6   B) 12   C) 14   D) 16
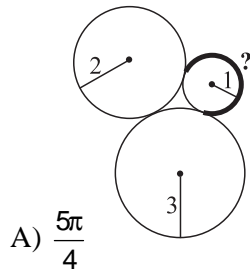 E) it depends on which sides of the big cube are red and which blue

23)

We note that $n!=1\cdot 2\cdot 3\cdot \ldots\cdot (n-1)\cdot n$. If $n!= 2^{15} \cdot 3^6 \cdot 5^3 \cdot 7^2 \cdot 11\cdot 13$, then $n =$

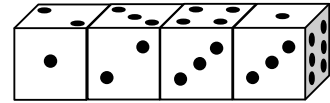 A) 13   B) 14   C) 15   D) 16   E) 17

24)

Perimeter ⟳ = ?

A) $\dfrac{5\pi}{4}$ B) $\dfrac{5\pi}{3}$ C) $\dfrac{\pi}{2}$ D) $\dfrac{3\pi}{2}$ E) $\dfrac{2\pi}{3}$

25)

Four identical dice are arranged in a row (see the fig.). The dice are not standard, i.e., the sum of points in the opposite faces of the dice not necessarily equals 7. Find the total sum of the points in all 6 touching faces of the dice.
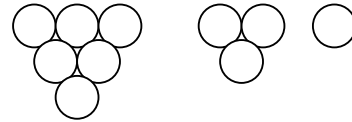
A) 19 B) 20 C) 21 D) 22 E) 23

26)

A 3-pyramid is a stack of the following 3 layers of balls:
In the same way we have a 4-pyramid, a 5-pyramid, etc.
All the balls anywhere on the outside of an 8-pyramid are black (balls are considered to be outside if they touch the circumscribed tetrahedron), the balls on the inside are all white. What kind of figure form the white balls?

A) 3-pyramid B) 4-pyramid C) 5-pyramid D) 6-pyramid E) 7-pyramid

27)

A square 4 x 4 table is divided into 16 unit squares (see the fig.) Find the maximum possible number of diagonals one can draw in these unit squares so that neither two of them have any common point (including endpoints).

A) 8 B) 9 C) 10 D) 11 E) 12

28)

A kangaroo always jumps 1 m or 3 m long. The kangaroo wants to go exactly 10 m. (We consider 1+3+3+3 and 3+3+3+1 as two different possibilities.) How many possibilities are there?
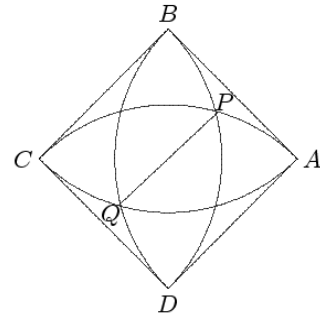
A) 28 B) 34 C) 35 D) 55 E) 56

29)

In the picture ABCD is a square of side 1 and the semicircles have centers on A, B, C and D. What is the length of PQ?

A) $2 - \sqrt{2}$ B) $\dfrac{3}{4}$ C) $\sqrt{5} - \sqrt{2}$

D) $\dfrac{\sqrt{3}}{3}$ E) $\sqrt{3} - 1$

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30)

How many 2007-digit numbers exist, in which every two-digit number composed of two sequential digit is devisable either by 17 or by 23?

A) 5   B) 6   C) 7   D) 9   E) more than 9

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GOOD LUCK !