3 point problems

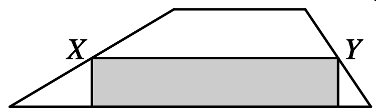
PROBLEM 01

A zebra crossing has alternate white and black stripes, each of width 50 cm. The crossing starts and ends with a white stripe and has 8 white stripes in all. What is the total width of the crossing?

(A) 7 m (B) 7.5 m (C) 8 m (D) 8.5 m (E) 9 m

PROBLEM 02

The area of the shaded rectangle is 13 cm^2 and X and Y are the midpoints of the sides of the trapezium.



What is the area of the trapezium?

(A) 24 cm^2 (B) 25 cm^2 (C) 26 cm^2 (D) 27 cm^2 (E) 28 cm^2

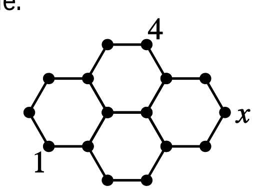
PROBLEM 03

Given that $P = 2 \times 3 + 3 \times 4 + 4 \times 5$, $Q = 2^2 + 3^2 + 4^2$ and $R = 1 \times 2 + 2 \times 3 + 3 \times 4$, which of the following statements is true?

(A) $Q < P < R$ (B) $P < Q = R$ (C) $P < Q < R$ (D) $R < Q < P$ (E) $Q = P < R$

PROBLEM 04

A number has to be written at each of the dots of the lattice shown so that the sum of the numbers at the ends of each line segment is the same.



Two of the numbers have already been written. What number goes in the place labelled x?

(A) 1 (B) 3 (C) 4 (D) 5 (E) more information is needed

PROBLEM 05

When 2011 was divided by a certain number, the remainder was 1011. Which of 100, 500 or 1000 was the divisor?

(A) 100 (B) 500 (C) 1000
(D) some other number (E) it is not possible to get this remainder

PROBLEM 06

A rectangular mosaic with area 360 cm^2 is made from square tiles, all the same size. The mosaic is 24 cm high and 5 tiles wide. What is the area of each tile?

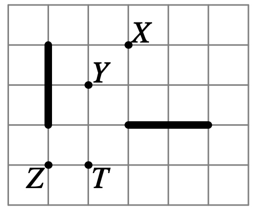
(A) 1 cm^2 (B) 4 cm^2 (C) 9 cm^2 (D) 16 cm^2 (E) 25 cm^2

PROBLEM 07

Every 4-digit number whose digits add up to 4 is listed in descending order. In which place in the list is the number 2011?

(A) 6th (B) 7th (C) 8th (D) 9th (E) 10th

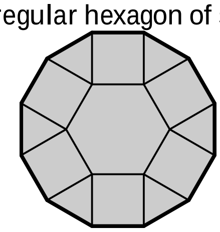
PROBLEM 08
Each of the two segments shown is a rotation of the other one.



Which of the marked points may be the centre of such a rotation?

(A) Only X (B) only X and Z (C) only X and T (D) only T (E) X, Y, Z and T

PROBLEM 09
The figure shows a shape consisting of a regular hexagon of side one unit, six triangles and six squares.



What is the perimeter of the shape?

(A) $6(1+\sqrt{2})$ (B) $6\left(1+\frac{\sqrt{3}}{2}\right)$ (C) 12 (D) $6+3\sqrt{2}$ (E) 9

PROBLEM 10
Three normal dice are piled on top of each other so that the total number of pips on two faces placed together is always 5. One of the visible faces on the bottom die has one pip. How many pips are on the top face on the top die?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

4 Point problems

PROBLEM 11
In a certain month there were 5 Mondays, 5 Tuesdays, and 5 Wednesdays. In the preceding month there were only 4 Sundays. Which of the following will the next month definitely include?

(A) exactly 4 Fridays (B) exactly 4 Saturdays (C) 5 Sundays (D) 5 Wednesdays (E) this situation is impossible

PROBLEM 12
Three sportsmen participated in a race: Michael, Fernando and Sebastian. Immediately after the start, Michael was in the lead, Fernando was second, and Sebastian third. During the race, Michael and Fernando changed places 9 times, Fernando and Sebastian did so 10 times, and Michael and Sebastian did so 11 times. In what order did they finish?

(A) Michael, Fernando, Sebastian (B) Fernando, Sebastian, Michael (C) Sebastian, Michael, Fernando (D) Sebastian, Fernando, Michael (E) Fernando, Michael, Sebastian

PROBLEM 13

Given that $9^n + 9^n + 9^n = 3^{2011}$, what is the value of n?

(A) 1005 (B) 1006 (C) 2010 (D) 2011 (E) none of them

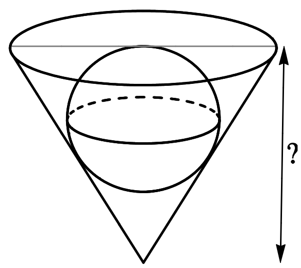
PROBLEM 14

Ulf has two cubes with sides of length a dm and $a + 1$ dm. The big cube is full of water and the small one is empty. Ulf fills the small cube by pouring some water from the big cube, leaving 217 litres in the big cube. How many litres of water was poured into the small cube?

(A) 243 (B) 512 (C) 125 (D) 1331 (E) 729

PROBLEM 15

A marble with radius 15 is rolled into a conical hole, which it just fits, as shown.

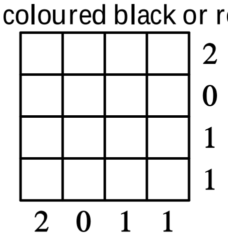


Viewed from the side the hole is an equilateral triangle. How deep is the hole?

(A) $30\sqrt{2}$ (B) $25\sqrt{3}$ (C) 45 (D) 60 (E) $60(\sqrt{3} - 1)$

PROBLEM 16

Each cell of the 4×4-grid shown is to be coloured black or red.



The number next to each row and column indicates how many cells in that row or column have to be black.
In how many ways can this be done?

(A) 0 (B) 1 (C) 3 (D) 5 (E) 9

PROBLEM 17

How many numbers are there in the longest run of consecutive 3-digit numbers, each of which has at least one odd digit?

(A) 1 (B) 10 (C) 110 (D) 111 (E) 221

PROBLEM 18

Nick wants to write an integer in each cell of the 3 × 3 grid shown, so that the sum of the numbers in every 2 × 2 square is 10.

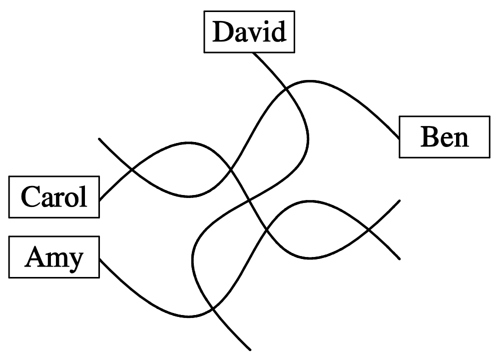
1		0
	2	
4		3

Five numbers have already been written. What is the sum of the four missing numbers?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

PROBLEM 19

During a rough sailing trip, Jane tried to sketch a map of her home village.

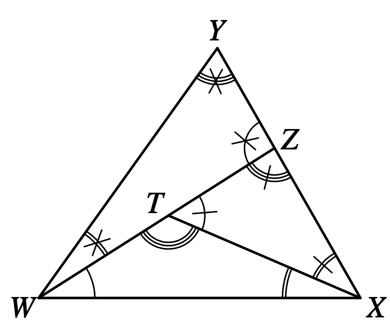


She managed to draw the four streets, their seven crossings and the houses of her friends. However, in reality Arrow Street, Nail Street and Ruler Street are all perfectly straight. The fourth street is Curvy Road. Who lives on Curvy Road?

(A) Amy (B) Ben (C) Carol (D) David (E) a better map is needed to be able to tell

PROBLEM 20

In triangle WXY, a point Z is chosen on the segment XY, then point T is chosen on the segment WZ, as shown.

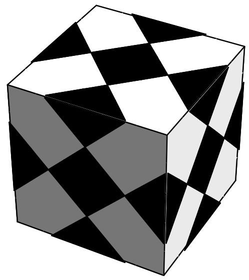


What is the smallest possible number of different values that the nine marked angles could take.

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

5 point problems

PROBLEM 21
Simon had a white plastic cube with edges of length 1 dm. He stuck several congruent black squares on the cube, as shown, so that the cube looked the same on every face.



What is the total black area?

(A) 37.5 cm^2 (B) 150 cm^2 (C) 225 cm^2 (D) 300 cm^2 (E) 375 cm^2

PROBLEM 22
A number is `cool' if it has five distinct digits and the first digit is equal to the sum of the other four digits. How many cool numbers are there?

(A) 72 (B) 144 (C) 168 (D) 216 (E) 288

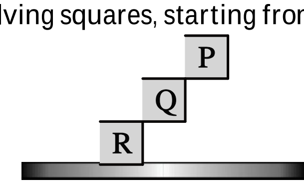
PROBLEM 23
The numbers x and y are both greater than 1. Which of the following fractions has the greatest value?

(A) $\frac{x}{y+1}$ (B) $\frac{x}{y-1}$ (C) $\frac{2x}{2y+1}$ (D) $\frac{2x}{2y-1}$ (E) $\frac{3x}{3y+1}$

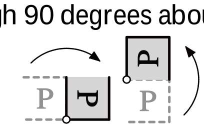
PROBLEM 24
A regular tetrahedron WXYZ has its face WXY in the plane P. The edge XY is on the line L. A different regular tetrahedron XYZT shares a face with WXYZ . Where does the line ZT meet P?

(A) on the same side of L as A, inside WXY
(B) on the same side of L as A, outside WXY
(C) on the opposite side of L to W
(D) ZT is parallel to P, so they do not meet
(E) the answer depends on the edge lengths of the two tetrahedra

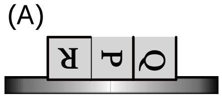
PROBLEM 25
Paul is playing a computer game involving squares, starting from the position shown.

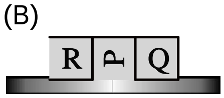


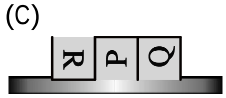
At each move, one square is turned through 90 degrees about a corner, as shown in the two examples.



The aim is to arrange the squares somewhere along the bottom of the screen.
Which of the following arrangements can Paul achieve?

(A)


(B)


(C)


(D)


(E) all of A-D are possible

PROBLEM 26

How many different ordered pairs of natural numbers (x, y) satisfy the equation $\frac{1}{x} + \frac{1}{y} = \frac{1}{3}$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

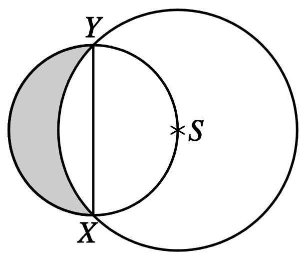
PROBLEM 27

For an integer $n \geq 2$ denote by $\langle n \rangle$ the biggest prime number which does not exceed n. How many positive integers k satisfy the equation $\langle k+1 \rangle + \langle k+2 \rangle = \langle 2k+3 \rangle$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than 3

PROBLEM 28

Two circles are constructed as shown in the figure.



The segment XY is a diameter of the smaller circle and the centre S of the larger circle lies on the smaller circle. The radius of the larger circle is r. What is the area of the shaded region?

(A) $\frac{\pi}{6}r^2$ (B) $\frac{\sqrt{3}\pi}{12}r^2$ (C) $\frac{1}{2}r^2$ (D) $\frac{\sqrt{3}}{4}r^2$ (E) another answer

PROBLEM 29

How many sets of four edges of a cube have the property that no two edges in the set have a common vertex?

(A) 6 (B) 8 (C) 9 (D) 12 (E) 18

PROBLEM 30

For which values of n, where $0 < n < 9$, is it possible to mark some cells in a 5×5 square in such a way that every 3×3 square contains exactly n marked cells?

(A) 1 (B) 1 and 2 (C) 1, 2 and 3 (D) 1, 2, 7 and 8 (E) all values from 1 to 8