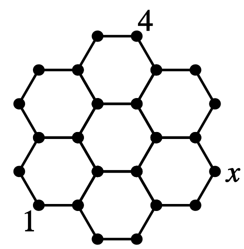
3 point problems

PROBLEM 01

A number has to be written at each of the dots of the lattice shown so that the sum of the numbers at the ends of each line segment is the same.



Two of the numbers have already been written. What number goes in the place labelled x?

(A) 1 (B) 3 (C) 4 (D) 5 (E) more information is needed

PROBLEM 02

Three sportsmen participated in a race: Michael, Fernando and Sebastian. Immediately after the start, Michael was in the lead, Fernando was second, and Sebastian third. During the race, Michael and Fernando changed places 9 times, Fernando and Sebastian did so 10 times, and Michael and Sebastian did so 11 times. In what order did they finish?

(A) Michael, Fernando, Sebastian (B) Fernando, Sebastian, Michael (C) Sebastian, Michael, Fernando (D) Sebastian, Fernando, Michael (E) Fernando, Michael, Sebastian

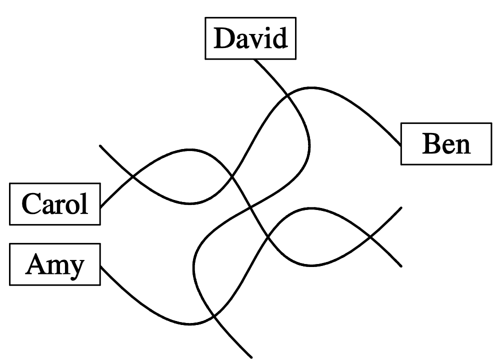
PROBLEM 03

Given that $2^x = 15$ and $15^y = 32$ what is the value of xy?

(A) 5 (B) $\log_2 15 + \log_{15} 32$ (C) $\log_2 47$ (D) 7 (E) $\sqrt{47}$

PROBLEM 04

During a rough sailing trip, Jane tried to sketch a map of her home village.



She managed to draw the four streets, their seven crossings and the houses of her friends. However, in reality Arrow Street, Nail Street and Ruler Street are all perfectly straight. The fourth street is Curvy Road. Who lives on Curvy Road?

(A) Amy (B) Ben (C) Carol (D) David (E) better map is needed to be able to tell

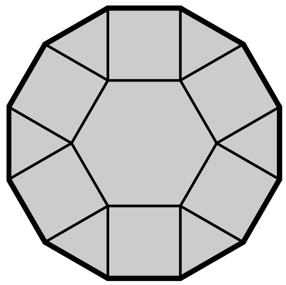
PROBLEM 05

Every 4-digit number whose digits add up to 4 is listed in descending order. In which place in the list is the number 2011?

(A) 6th (B) 7th (C) 8th (D) 9th (E) 10th

PROBLEM 06

The figure shows a shape consisting of a regular hexagon of side one unit, six triangles and six squares.

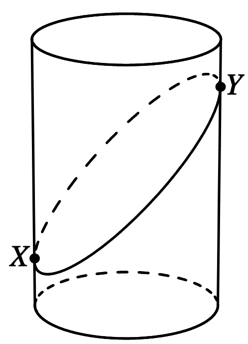


What is the perimeter of the shape?

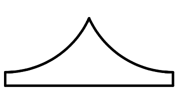
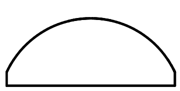
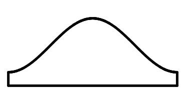
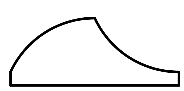
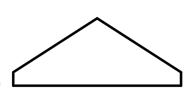
(A) $6(1+\sqrt{2})$ (B) $6\left(1+\frac{\sqrt{3}}{2}\right)$ (C) 12 (D) $6+3\sqrt{2}$ (E) 9

PROBLEM 07

A rectangular piece of paper is wrapped around a cylinder and a plane cut is made through the cylinder and paper. The cut passes through the points X and Y shown in the figure.

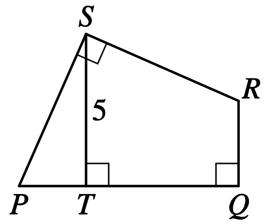


The bottom part of the paper is then unwrapped. Which picture could be the result?

(A)  (B)  (C)  (D)  (E) 

PROBLEM 08

The figure shows a quadrilateral $PQRS$, in which $PS = SR$, $\measuredangle PSR = \measuredangle PQR = 90°$, $ST \perp PQ$ and $ST = 5$.



What is the area of the quadrilateral $PQRS$?

(A) 20 (B) 22.5 (C) 25 (D) 27.5 (E) 30

PROBLEM 09

Andrew wrote all the odd numbers from 1 to 2011 on a board and then Bob erased all the multiples of 3. How many numbers were left on the board?

(A) 335 (B) 336 (C) 671 (D) 1005 (E) 1006

PROBLEM 10

Max and Hugo throw a handful of dice to decide who shall be the first to jump into a cold lake. If there are no sixes it will be Max; if there is one six it will be Hugo; and if there are two or more sixes they will not take a swim that day. How many dice should they throw if they want there to be an equal chance of having to jump in first?

(A) 3 (B) 5 (C) 8 (D) 9 (E) 17

4 point problems

PROBLEM 11

Three rectangles are to be combined, without gaps or overlaps, to form a large rectangle. One of the three has size 7 by 11 and another has size 4 by 8. The third rectangle is chosen to have the largest possible area. What is its size?

(A) 1 by 11 (B) 3 by 4 (C) 3 by 8 (D) 7 by 8 (E) 7 by 11

PROBLEM 12

Mike wants to write integers in the cells of the 3 × 3 grid shown so that the sum of the numbers in every 2 × 2 square equals 10.

	2	
1		3
	4	

Four numbers have already been written in the grid.
Which of the following values could be the sum of the missing five numbers?

(A) 9 (B) 10 (C) 12 (D) 13 (E) none of A-D is possible

PROBLEM 13

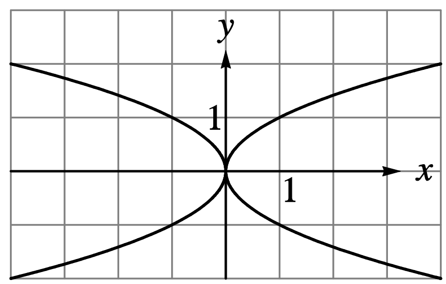
48 children went on a ski trip. Six of them had exactly one sibling on the trip, nine children went with exactly two siblings and four of them went with exactly three siblings. The rest of the children didn't have any siblings on the trip. How many families had someone on the trip?

(A) 19 (B) 25 (C) 31 (D) 36 (E) 48

PROBLEM 14

How many of the graphs of the functions

$y = x^2, y = -x^2, y = \sqrt{x}, y = -\sqrt{x}, y = \sqrt{-x}, y = -\sqrt{-x}, y = \sqrt{|x|}, y = -\sqrt{|x|}$

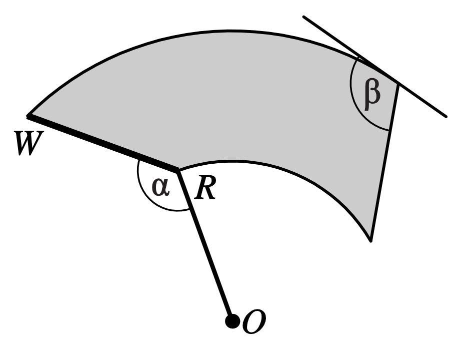


are included in the figure?

(A) none (B) 2 (C) 4 (D) 6 (E) all 8

PROBLEM 15

The rear windshield wiper of a car is constructed in such a way that the wiper blade RW and the connecting rod OR are of equal length and joined at a fixed angle α. The wiper pivots about the centre O and clears the shaded area shown.

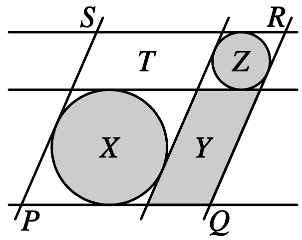


Determine the angle β between the right-hand edge of the cleared area and the tangent to the curved upper edge.

(A) $\frac{3\pi-\alpha}{2}$ (B) $\pi - \frac{\alpha}{2}$ (C) $\frac{3\pi}{2} - \alpha$ (D) $\frac{\pi}{2} + \alpha$ (E) $\pi + \frac{\alpha}{2}$

PROBLEM 16

The figure shows three horizontal lines and three parallel slanting lines.

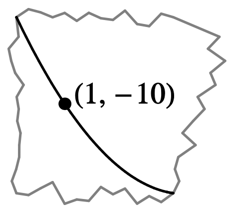


Each circle is tangent to four of the lines. The areas of the shaded figures are X, Y and Z, as shown, and W is the area of the parallelogram $PQRS$. What is the smallest number of the areas X, Y, Z and W that need to be known in order to be able to calculate the area T of the parallelogram indicated?

(A) 1 (B) 2 (C) 3 (D) 4 (E) T cannot be calculated from X, Y, Z and W

PROBLEM 17

On the (x, y)-plane, with the axes positioned in the standard way, the point $A(1, -10)$ was marked on the parabola $y = ax^2 + bx + c$. After that, the coordinate axes and almost all of the parabola were erased, leaving the figure shown.



Which of the following statements can be false?

(A) $a > 0$ (B) $b < 0$ (C) $a + b + c < 0$ (D) $b^2 > 4ac$ (E) $c < 0$

PROBLEM 18

The sides PQ, QR, RS, ST, TU and UP of a hexagon are all tangent to a common circle. The lengths of the sides PQ, QR, RS, ST and TU are 4, 5, 6, 7 and 8 respectively. What is the length of side UP?

(A) 9 (B) 8 (C) 7 (D) 6 (E) the length cannot be calculated from this information

PROBLEM 19

Find the sum of all positive integers x less than 100 for which $x^2 - 81$ is a multiple of 100.

(A) 200 (B) 100 (C) 90 (D) 81 (E) 50

PROBLEM 20

The brothers Andrej and Brano gave truthful answers to a question about how many members their chess club has. Andrej said: ``All the members of our club, except for five girls, are boys." Brano said: ``Every group of six members always includes at least four girls." How many members does their chess club have?

(A) 6 (B) 7 (C) 8 (D) 12 (E) 18

5 point problems

PROBLEM 21

There are some balls in a raffle bucket. One positive integer is written on each ball; all the integers are different. A number divisible by 6 is written on 30 balls, a number divisible by 7 is written on 20 balls and a number divisible by 42 is written on 10 balls. What is the smallest possible number of balls in the bucket?

(A) 30 (B) 40 (C) 53 (D) 54 (E) 60

PROBLEM 22

Consider the two arithmetic sequences $5, 20, 35, \ldots$, and $35, 61, 87, \ldots$. How many different arithmetic sequences of positive integers are there which have each of the given sequences as a subsequence?

(A) 1 (B) 3 (C) 5 (D) 26 (E) infinitely many

PROBLEM 23

The sequence of numerical functions $f_1(x), f_2(x), \ldots$, satisfies the following two conditions:

$f_1(x) = x$; and $f_{n+1}(x) = \frac{1}{1-f_n(x)}$. What is the value of $f_{2011}(2011)$?

(A) 2011 (B) $-\frac{1}{2010}$ (C) $\frac{2010}{2011}$ (D) 1 (E) – 2011

PROBLEM 24

A box contains some red balls and some green balls. If we randomly choose two balls from the box, the probability that they are the same colour is $\frac{1}{2}$. Which of the following could be the total number of balls in the box?

(A) 81 (B) 101 (C) 1000 (D) 2011 (E) 10001

PROBLEM 25

An airline company does not charge luggage fees if the luggage is under a certain weight limit. For every extra kilogram, a fee is charged. The luggage of Mr and Mrs Trip weighed 60 kg and they paid 3 euros. Mr Wander's luggage weighed the same but he paid 10.50 euros. What is the maximum weight of luggage one passenger can take free of charge?

(A) 10 kg (B) 18 kg (C) 20 kg (D) 25 kg (E) 39 kg

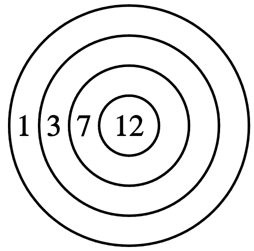
PROBLEM 26

In the expression $\frac{K \cdot A \cdot N \cdot G \cdot A \cdot R \cdot O \cdot O}{G \cdot A \cdot M \cdot E}$ different letters stand for different non-zero digits, but the same letter always stands for the same digit. What is the smallest possible positive integer value of the expression?

(A) 1 (B) 2 (C) 3 (D) 5 (E) 7

PROBLEM 27

Robin Hood shoots three arrows at a target, earning points for each shot as shown in the figure. All the arrows hit the target.



How many different point totals can he obtain in this way?

(A) 13 (B) 17 (C) 19 (D) 20 (E) 21

PROBLEM 28

Let a, b and c be positive integers such that $a^2 = 2b^3 = 3c^5$. What is the minimum possible number of divisors of abc (including 1 and abc)?

(A) 30 (B) 49 (C) 60 (D) 77 (E) 1596

PROBLEM 29

Twenty different positive integers are written in a 4×5 table. Any two neighbours (numbers in cells with a common side) have a common divisor greater than 1. If n is the largest number in the table, find the smallest possible value of n.

(A) 21 (B) 24 (C) 26 (D) 27 (E) 40

PROBLEM 30

A $3 \times 3 \times 3$ cube is composed of 27 identical small cubes. A plane is perpendicular to a diagonal of the large cube and passes through its centre. How many small cubes does that plane intersect?

(A) 17 (B) 18 (C) 19 (D) 20 (E) 21