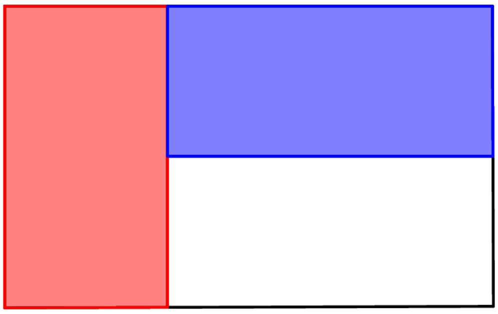
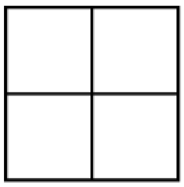
SECTION ONE - (3 point problems)

1. The flag of Kangoraland is a rectangle which is divided into three smaller equal rectangles as shown. What is the ratio of the side lengths of the white rectangle?



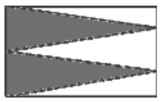
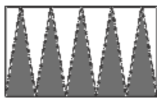
(A) 1: 2 **(B)** 2: 3 **(C)** 2: 5
(D) 3: 7 **(E)** 4: 9

2. The numbers 1, 2, 3 and 4 are each written in different cells of the 2 × 2 table. After that, the sum of the numbers in each row and column is calculated. Two of these sums are 4 and 5. What are the other two sums?



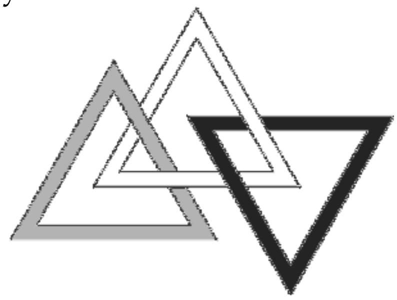
(A) 6 and 6 **(B)** 3 and 5 **(C)** 4 and 5
(D) 4 and 6 **(E)** 5 and 6

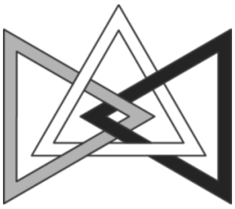
3. A rectangle has been shaded in five different ways as shown. In which diagram does the shaded part have the largest area?

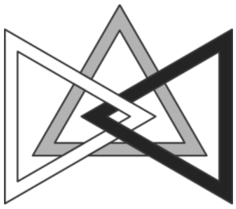
(A)  **(B)**  **(C)** 

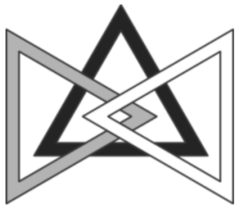
(D)  **(E)** 

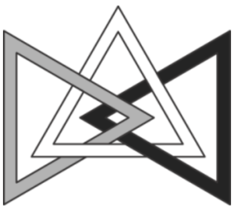
4. Three triangles are linked as shown. Which of the following pictures shows these three triangles linked in the same way?

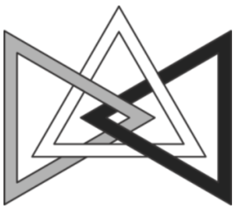


(A) 

(B) 

(C) 

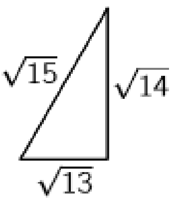
(D) 

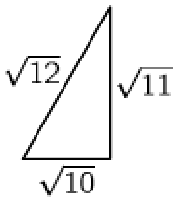
(E) 

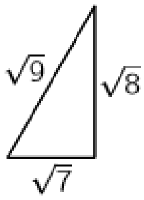
5. A pyramid has 23 triangular faces. How many edges does this pyramid have?

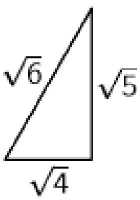
(A) 23 **(B)** 24 **(C)** 46
(D) 48 **(E)** 69

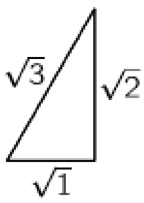
6. The following sketches suggest right-angled triangles. Which one is indeed right-angled?

(A) 

(B) 

(C) 

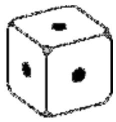
(D) 

(E) 

7. What is the first (leftmost) digit of the smallest positive integer whose digits add up to 2019?

(A) 2 **(B)** 3 **(C)** 4
(D) 5 **(E)** 6

8. Each of the faces of a die is marked with either 1, 2 or 3 dots so that the probability of rolling a 1 is $\frac{1}{2}$, the probability of rolling a 2 is $\frac{1}{3}$ and the probability of rolling a 3 is $\frac{1}{6}$. Which of the following cannot be a view of this die?

(A)  **(B)**  **(C)** 

(D)  **(E)** 

9. Michael invented a new "$\Diamond$" operation for real numbers, defined as $x \Diamond y = y - x$. If $a, b,$ and c satisfy $(a\Diamond b)\Diamond c = a\Diamond(b\Diamond c)$, which of the following statements is necessarily true?

(A) $a = b$ **(B)** $b = c$ **(C)** $a = c$
(D) $a = 0$ **(E)** $c = 0$

10. How many of the numbers from 2^{10} to 2^{13}, inclusive, are divisible by 2^{10}?

(A) 2 **(B)** 4 **(C)** 6
(D) 8 **(E)** 16

SECTION TWO - (4 point problems)

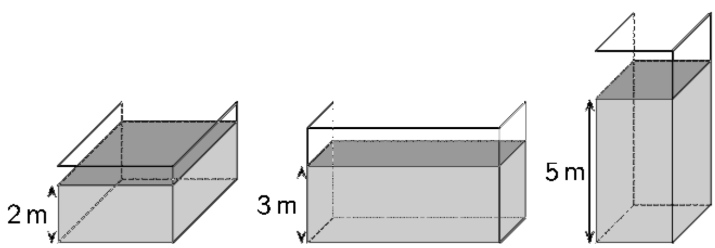
11. Which is the highest power of 3 dividing the number $7! + 8! + 9!$?

(A) 3^2 **(B)** 3^4 **(C)** 3^5
(D) 3^6 **(E)** a power of 3 higher than 3^6

12. This year, the number of boys in my class has increased by 20% and the number of girls has decreased by 20%. We now have one student more than before. Which of the following could be the number of students in my class now?

(A) 22 **(B)** 26 **(C)** 29
(D) 31 **(E)** 34

13. A container in the shape of a rectangular box is partially filled with 120 m^3 of water. The depth of the water is either 2 m or 3 m or 5 m, depending on which side of the box is on the ground, as shown (not to scale). What is the volume of the container?

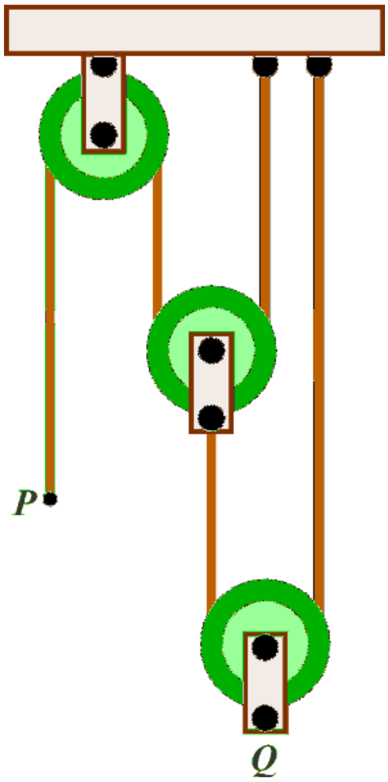


(A) 160 m^3 **(B)** 180 m^3 **(C)** 200 m^3
(D) 220 m^3 **(E)** 240 m^3

14. Three kangaroos, Alex, Bob and Carl, go for a walk every day. If Alex doesn't wear a hat, then Bob wears a hat. If Bob doesn't wear a hat, then Carl wears a hat. Today Carl is not wearing a hat. Who is certainly wearing a hat today?

(A) only Alex and Bob
(B) only Alex
(C) Alex, Bob and Carl
(D) neither Alex nor Bob
(E) only Bob

15. The system shown consists of three pulleys with vertical sections of rope between them. The end P is moved down 24 centimeters. How many centimeters does point Q move up?



(A) 24
(B) 12
(C) 8
(D) 6
(E) $\frac{24}{5}$

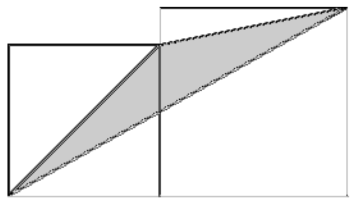
16. A positive integer n is called *good* if its largest divisor (excluding n) is equal to $n-6$. How many *good* positive integers are there?

(A) 1
(B) 2
(C) 3
(D) 6
(E) infinitely many

17. A box contains 4 chocolates and 1 fruit chew. John and Mary take turns drawing a treat out of the box without replacement. Whoever draws the fruit chew wins. John draws first. What is the probability that Mary wins?

(A) $\frac{2}{5}$
(B) $\frac{3}{5}$
(C) $\frac{1}{2}$
(D) $\frac{5}{6}$
(E) $\frac{1}{3}$

18. Two adjacent squares with side lengths a and b ($a < b$) are shown. What is the area of the shaded triangle?



(A) $\sqrt{ab}$ **(B)** $\frac{1}{2}a^2$ **(C)** $\frac{1}{2}b^2$

(D) $\frac{1}{4}(a^2+b^2)$ **(E)** $\frac{1}{2}(a^2+b^2)$

19. What is the integer part of

$$\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20+\sqrt{20}}}}}?$$

(A) 4 **(B)** 5 **(C)** 6

(D) 20 **(E)** 25

20. To calculate the result of $\frac{a+b}{c}$, Sara types $a+b\div c =$ on a calculator and the result is 11 (a, b, and c are positive integers). She then types $b+a\div c =$ and she is surprised to see that the result is 14. She realizes that the calculator is designed to calculate divisions before additions. What is the correct result of $\frac{a+b}{c}$?

(A) 1 **(B)** 2 **(C)** 3

(D) 4 **(E)** 5

SECTION THREE - (5 point problems)

21. Let a be the sum of all positive divisors of 1024 and b the product of all positive divisors of 1024. Then

(A) $(a-1)^5=b$ **(B)** $(a+1)^5=b$ **(C)** $a^5=b$

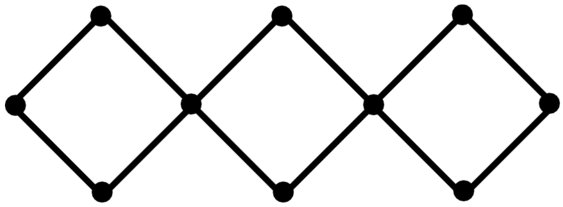
(D) $a^5-1=b$ **(E)** $a^5+1=b$

22. What is the set of all values of the parameter a for which the number of solutions of the equation $2-|x|=ax$ is equal to two?

(A) $(-\infty,-1]$ **(B)** $(-1,1)$ **(C)** $[1,+\infty)$

(D) $\{0\}$ **(E)** $\{-1,1\}$

23. The vertices of the network shown are labelled with the numbers from 1 to 10. The sum S of the four labels on each square is the same. What is the least possible value of S?



(A) 18 **(B)** 19 **(C)** 20
(D) 21 **(E)** 22

24. How many planes pass through at least three vertices of a given cube?

(A) 6 **(B)** 8 **(C)** 12
(D) 16 **(E)** 20

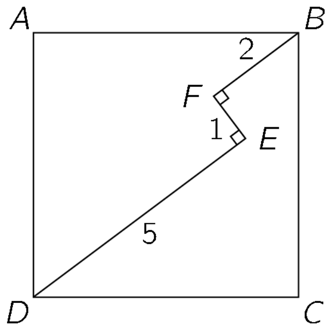
25. Four distinct straight lines pass through the origin of the coordinate system. They intersect the parabola $y = x^2 - 2$ at eight points. What can be the product of the x-coordinates of these eight points?

(A) only 16 **(B)** only -16 **(C)** only 8
(D) only -8 **(E)** There are several possible products.

26. For how many integers n is $|n^2 - 2n - 3|$ a prime number?

(A) 1 **(B)** 2 **(C)** 3
(D) 4 **(E)** infinitely many

27. A path $DEFB$ with $DE \perp EF$ and $EF \perp FB$ lies inside the square $ABCD$ as shown. Given that $DE = 5$, $EF = 1$ and $FB = 2$, what is the length of the side of the square?



(A) $3\sqrt{2}$ **(B)** $\frac{7\sqrt{2}}{2}$ **(C)** $\frac{11}{2}$
(D) $5\sqrt{2}$ **(E)** none of the previous

Time Allowed: **150 minutes**

28. The sequence $a_1, a_2, a_3, \ldots$ starts with $a_1 = 49$. For $n \geq 1$, the number a_{n+1} is obtained by adding 1 to the sum of the digits of a_n and then squaring the result. Thus $a_2 = (4+9+1)^2 = 196$. Determine a_{2019}.

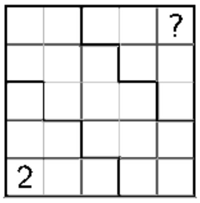
(A) 121 **(B)** 25 **(C)** 64
(D) 400 **(E)** 49

29. Three different numbers are chosen at random from the set $\{1,2,3,\ldots,10\}$. What is the probability that one of them is the average of the other two?

(A) $\frac{1}{10}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$

(D) $\frac{1}{3}$ **(E)** $\frac{1}{2}$

30. The square shown is filled with numbers in such a way that each row and each column contains the numbers 1, 2, 3, 4 and 5 exactly once. Moreover, the sum of the numbers in each of the three bold-bordered areas is equal. What number is in the upper right corner?



(A) 1 **(B)** 2 **(C)** 3
(D) 4 **(E)** 5

-- Good Luck --