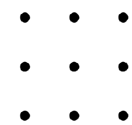
Junior Level: Class (9 & 10) Max Time: 2 Hours

3-Point Problems

1. Which among these numbers is multiple of 3?

(**A**) 2009 (**B**) $2+0+0+9$ (**C**) $(2+0)\cdot(0+9)$
(**D**) 2^9 (**E**) $200-9$

2. Which minimal number of the points in the figure one need to remove so that no 3 of the remaining points are collinear(lie on the same straight line)?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 7

3. In a popular race have participated 2009 people. The number of people that John has won is triple than the number of people that had won to John. In what place has been classified John in the race?

(**A**) 503 (**B**) 501 (**C**) 500 (**D**) 1503 (**E**) 1507

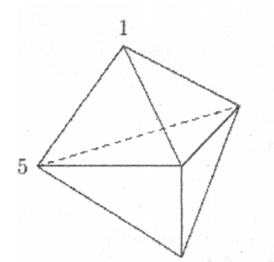
4. What is the value of the $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of 1000?

(**A**) 250 (**B**) 200 (**C**) 100
(**D**) 50 (**E**) None of these

5. A long sequence of digits has been composed by writing the number 2009 repeatedly 2009 times. The sum of those odd digits in the sequence that are immediately followed by an even digit is equal to

(**A**) 2 (**B**) 9 (**C**) 4018 (**D**) 18072 (**E**) 18081

6. The picture shows a solid formed with 6 triangular faces. At each vertex there is a number. For each face we consider the sum of the 3 numbers at the vertices of that face. If all the sums are the same and two of the numbers are 1 and 5 as shown, what is the sum of all the 5 numbers?

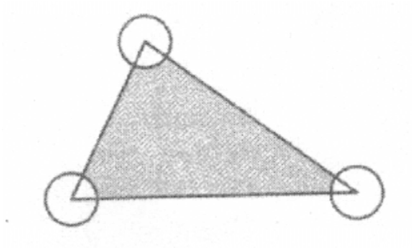


(**A**) 9 (**B**) 12 (**C**) 17 (**D**) 18 (**E**) 24

7. How many positive integers have equally many digits in the decimal representation of their square and their cube?

(**A**) 0 (**B**) 3 (**C**) 4
(**D**) 9 (**E**) infinitely many

8. The area of the triangle of the picture is 80 m^2 and the radius of the circles centered at the vertices is 2 m.



What is the measure, in m^2, of the shaded area?

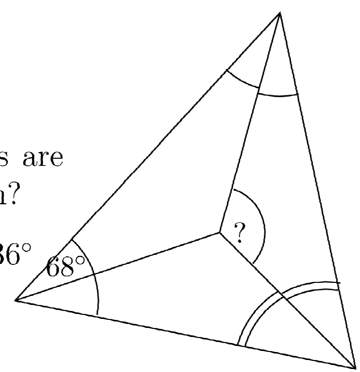
(**A**) 76 (**B**) $80 - 2\pi$ (**C**) $40 - 4\pi$ (**D**) $80 - \pi$ (**E**) 78π

9. Leonard has written a sequence of numbers, such that each number (from the third number in the sequence) was a sum of previous two numbers in the sequence. The forth number in the sequence was 6 and the sixth number in the sequence was 15. What was the seventh number in the sequence?

(**A**) 9 (**B**) 16 (**C**) 21 (**D**) 22 (**E**) 24

10. A triangle has an angle of 68°. The three angle bisectors are drawn. How many degrees is the angles with the question sign?

(**A**) 120° (**B**) 124° (**C**) 128° (**D**) 132° (**E**) 136°

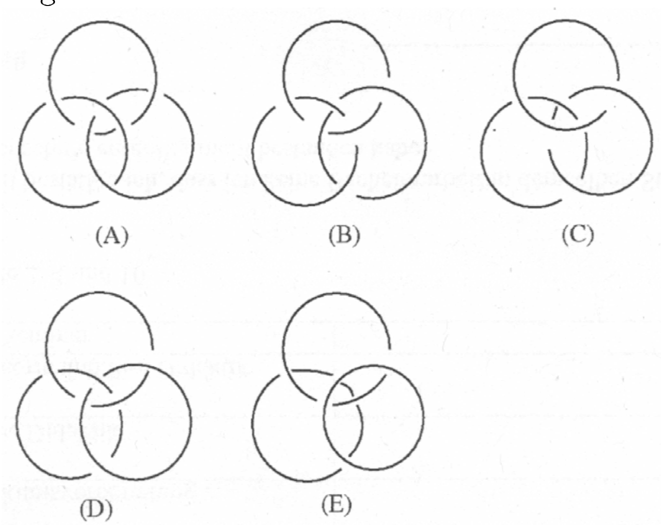


4-Point Problems

11. At each test, the mark can be 0, 1, 2, 3, 4 or 5. After 4 tests, Mary's average is 4. One of the sentences cannot be true. Which is it?

(**A**) Mary got only the mark 4.
(**B**) Mary got the mark 3 exactly twice.
(**C**) Mary got the mark 3 exactly 3 times.
(**D**) Mary got the mark 1 exactly once.
(**E**) Mary got the mark 4 exactly twice.

12. The Borromean rings have the surprising property that the three of them cannot be separated without destroying them but once one of them is removed (regardless which one), the other two are not linked anymore. Which of the following figures shows the Borromean rings?



(**A**) A (**B**) B (**C**) C (**D**) D (**E**) E

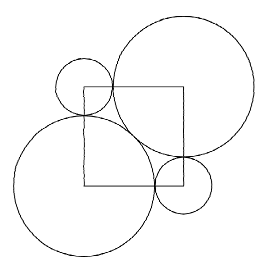
13. On the island of nobles and liars 25 people are standing in a queue. Everyone, except the first person in the queue, said, that the person before him in the queue is a liar, and the first man in the queue said, that all people, standing after him are liars. How many liar are there in the queue? (Nobles always speak the truth, and liars always tell lies.)

(**A**) 0 (**B**) 12 (**C**) 13
(**D**) 24 (**E**) impossible to determine

14. If $a\square b = ab + a + b$, and $3\square 5 = 2\square x$ than x equal:

(**A**) 3 (**B**) 6 (**C**) 7 (**D**) 10 (**E**) 12

15. Around the vertices of a square circles are drawn: 2 large and 2 small ones. The large circles are tangent to each other and to both the small circles. The radius of a large circle $= \ldots \cdot$ the radius of a small circle.



(**A**) $\frac{2}{9}$ (**B**) $\sqrt{5}$ (**C**) $1+\sqrt{2}$ (**D**) 2.5 (**E**) $0,8\pi$

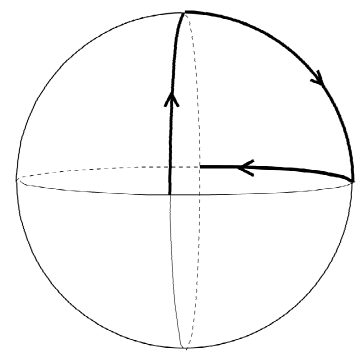
16. The difference between $\sqrt{n}$ and 10 is less than 1. How many such n integer exist?

(**A**) 19 (**B**) 20 (**C**) 39 (**D**) 40 (**E**) 41

17. Man Friday wrote down in a row several different natural numbers not exceeding 10. Robinson Crusoe examined these numbers and noticed with satisfaction that in each pair of neighbouring numbers one of the numbers is divisible by another. At most how many numbers did Man Friday write down?

(**A**) 6 (**B**) 7 (**C**) 8 (**D**) 9 (**E**) 10

18. 3 circular hoops are joined together so that they intersect at right angles as shown. A ladybird lands on an intersection and crawls around the hoop as follows : she travels along a quarter-circle, turns right 90°, travels along a quarter-circle and turns left 90°. Proceeding in this way, how many quarter-circles will she travel along before she first returns to her starting point?



(**A**) 6 (**B**) 9 (**C**) 12 (**D**) 15 (**E**) 18

19. How many zeros should be inscribed in place of $*$ in the decimal fraction $1.*1$ in order to get a number that is less than $\frac{2009}{2008}$ but greater than $\frac{20009}{20008}$?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

20. If $a = 2^{25}$, $b = 8^8$ and $c = 3^{11}$, then

(**A**) $a < b < c$ (**B**) $b < a < c$ (**C**) $c < b < a$ (**D**) $c < a < b$ (**E**) $b < c < a$

5-Point Problems

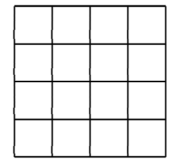
21. How many ten-digit numbers only composed of 1, 2 and 3 exist, in which any two neighboring digits differ by 1.

(**A**) 16 (**B**) 32 (**C**) 64 (**D**) 80 (**E**) 100

22. Young Kangaroo has 2009 unit $1 \times 1 \times 1$ cubes that he has placed forming a cuboid. He has also 2009 stickers 1×1 that he must use to colouring the outer surface of cuboid. Young Kangaroo has achieved its goal and it left stickers. How many stickers have left?

(**A**) more than 1000 (**B**) 763
(**C**) 476 (**D**) 49
(**E**) It is not true that the Kangaroo can achieve his goal

23. Bob wants to place draughts into cells of 4×4 board so that the numbers of the draughts in any row and in any column will be different (more than one draught can be placed into one cell as well as the cell can be empty). What is the smallest possible number of the draught placed on the board?



(**A**) 21 (**B**) 22 (**C**) 23 (**D**) 24 (**E**) 25

24. Some oranges, peaches, apples and bananas were put in a row, so that somewhere in the row each type of fruit can be found next to each other type of fruit. What is the least number of fruits in the row?

(**A**) 4 (**B**) 5 (**C**) 8
(**D**) 11 (**E**) this situation is impossible

25. What is the least integer n, for which $(2^2 - 1) \cdot (3^2 - 1) \cdot (4^2 - 1) \cdot \ldots (n^2 - 1)$ is a perfect square?

(**A**) 6 (**B**) 8 (**C**) 16 (**D**) 27 (**E**) other answer

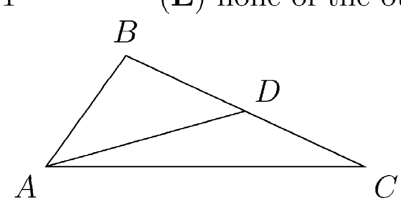
26. All divisions of the number N, not equal to N and to 1, were written in line. It occurred, that the greatest of the divisors in the line is 45 times as great as the least one. How many numbers satisfy this condition?

(**A**) 0 (**B**) 1 (**C**) 2
(**D**) more than 2 (**E**) impossible to determine

27. A kangaroo is sitting in the origin of a coordinate system. It can jump 1 unit vertically or horizontally. How many points are there in the plane where the kangaroo can be after 10 jumps?

(**A**) 121 (**B**) 100 (**C**) 400 (**D**) 441 (**E**) none of the others

28. Let AD be a median in the triangle ABC. The angle ACB has measure $30°$, the angle ADB has measure $45°$. What is the measure of the angle BAD?



(**A**) $45°$ (**B**) $30°$ (**C**) $25°$ (**D**) $20°$ (**E**) $15°$

29. Find the minimal quantity of numbers one should remove from the set $\{1, 2, 3, \ldots 16\}$ such that the sum of any 2 remaining numbers is a perfect square.

(**A**) 10 (**B**) 9 (**C**) 8 (**D**) 7 (**E**) 6

30. A prime number is defined as being "strange" if it is either a one digit prime or if it has two or more digits but both numbers obtained by omitting either its first or its last digit is also "strange". How many strange primes are there?

(**A**) 6 (**B**) 7 (**C**) 8 (**D**) 9 (**E**) 11