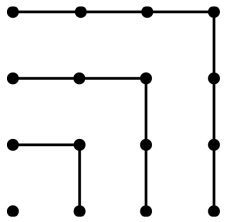
International Kangaroo Mathematics Contest 2010

Student Level: Class (11 & 12) **Max Time: 3 Hours**

3-point *problems*



Q1) Using next picture we can observe that
1+3+5+7 = 4 x 4.
What is the value of 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17?

A) 14 x 14 B) 9 x 9 C) 4 x 4 x 4 D) 16 x 16 E) 7 x 9

Q2) If both rows have the same sum, what is the value of *?

1	2	3	4	5	6	7	8	9	10	2010
11	12	13	14	15	16	17	18	19	20	*

A) 1010 B) 1020 C) 1910 D) 1990 E) 2000

Q3) Two empty cubes have base areas of 1 dm^2 and 4 dm^2 respectively. We want to fill the bigger cube with spring water which we fetch using the smaller cube. How many times do we have to go to the spring?

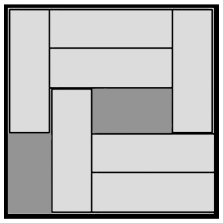
A) 2 times B) 4 times C) 6 times D) 8 times E) 16 times

Q4) How many four-digit numbers formed of only odd digits are divisible by five?

A) 900 B) 625 C) 250 D) 125 E) 100

Q5) The director of a company said: *"Each of our employees is at least 25 years old."* Later, it turned out, that he was not right. It means, that

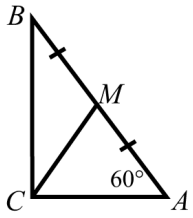
A) all employees in the company are exactly 25 years old.;
B) all employees in the company are more than 26 years old;
C) none of the employees in the company is 25 years old yet;
D) some employee in the company is less than 25 years old;
E) some employee in the company is exactly 26 years old.



Q6) There are seven 3×1 bars in the box as showing the figure. We wish to slide some bars in the box so there will be room for one more bar? At least how many bars must be moved in that case?

A) 2 B) 3 C) 4 D) 5 E) It is impossible

Q7) The triangle *ABC* is right-angled, *M* is the midpoint of the hypotenuse *AB* and $\angle A = 60°$. $\angle BMC$ =



A) 105° B) 108° C) 110° D) 120° E) 125°

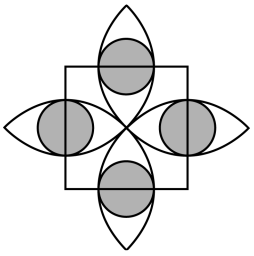
Q8) Choose a number which could be equal to the number of edges of some prism.

A) 100 B) 200 C) 2008 D) 2009 E) 2010

Q9) How many 2-digit numbers *xy* have digits *x* and *y* with the property $(x-3)^2 + (y-2)^2 = 0$?

A) 1 B) 2 C) 6 D) 32 E) none

Q10) In the picture, the side of the square has length 2, the semicircles go through the center of the square and have centers on the vertices of the square. The shaded circles have centers on the sides of the squares and are tangent to the semicircles. What is the shaded area?



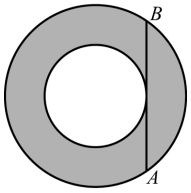
A) $4(3-2\sqrt{2})\pi$ B) $\sqrt{2}\,\pi$ C) $\frac{\sqrt{3}}{4}\pi$ D) π E) $\frac{1}{4}\pi$

4-point *problems*

Q11) The three numbers $\sqrt{7}$, $\sqrt[3]{7}$, $\sqrt[6]{7}$ are consecutive terms of a geometric progression.The next term of the progression is

A) $\sqrt[9]{7}$ B) $\sqrt[12]{7}$ C) $\sqrt[5]{7}$ D) $\sqrt[10]{7}$ E) 1

Q12) The chord *AB* is tangent to the smaller of the concentric circle. If *AB* = 16, what is the area of the shaded region?

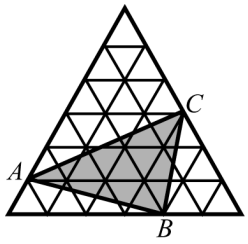


A) 32π B) 63π C) 64π D) $32\pi^2$ E) it depends on radii of the circles

Q13) The integer numbers x and y satisfy $2x = 5y$. Only one of the following can be $x + y$. Which is it?

A) 2011 B) 2010 C) 2009 D) 2008 E) 2007

Q14) The big equilateral triangle consists of 36 smaller equilateral triangles with area 1 cm^2 each. Find the area of ΔABC.



A) 11 cm^2 B) 12 cm^2 C) 13 cm^2 D) 14 cm^2 E) 15 cm^2

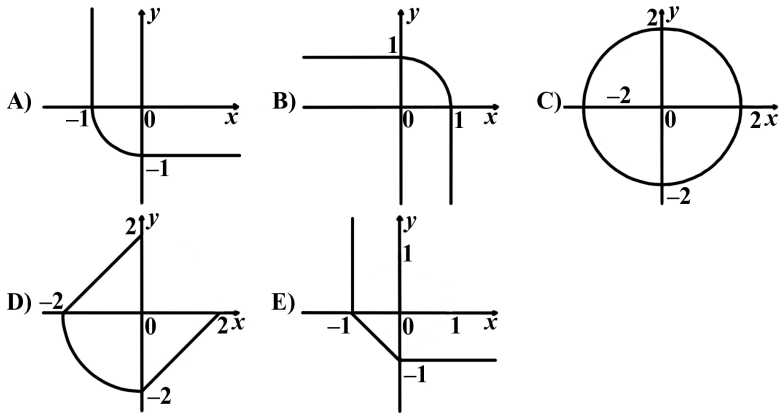
Q15) There are balls of three colours in a bag: blue, green and red (there is at least one of each colour). We know, that in case we are blindfolded and draw five balls randomly, there will definitely be at least two red ones and at least three will be the same colour. How many blue balls are there in the bag?

A) 1 B) 2 C) 3 D) 4

E) It is impossible to find out without more detailed information.

Q16) Which of these graphs corresponds with the set of all solutions of the equation

$$(x - |x|)^2 + (y - |y|)^2 = 4?$$



Q17) How many right-angled triangles can be formed by joining three vertices of a given regular 14-gon?

A) 42 B) 84 C) 88 D) 98 E) 168

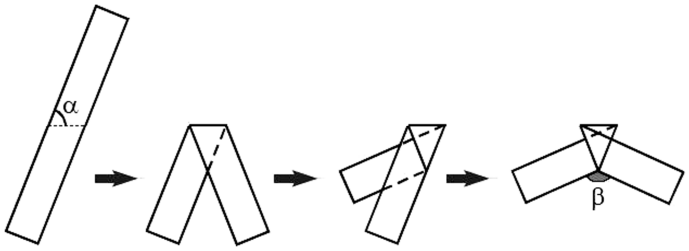
Q18) Each star in the expression 1 * 2 * 3 * 4 * 5 * 6 * 7 * 8 * 9 * 10 is replaced either by "+" or "·". Let N be the largest possible value of the expression that can be obtained this way. What is the smallest prime factor of N?

A) 2 B) 3 C) 5 D) 7 E) some other number

Q19) The lengths of the sides of a triangle in centimeters are the natural numbers 13, x and y. Find the perimeter if $xy = 105$.

A) 35 B) 39 C) 51 D) 69 E) 119

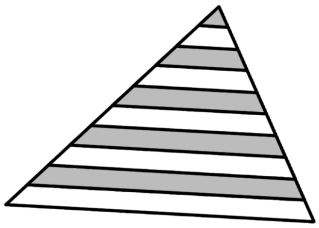
Q20) The paper ribbon is folded three times as shown. Find β if α = 70º.



A) 140º B) 130º C) 120º D) 110º E) 100º

5-point *problems*

Q21) Lines parallel to the base divide each of the other two sides of the triangle shown into 10 equal segment.
Which percentage of the area of triangle is grey?



A) 42.5% B) 45% C) 46% D) 47.5% E) 50%

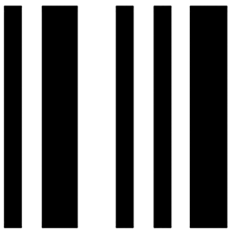
Q22) 100 people took part in a race, no two of which arrived at the same time. Each was asked, in which place he/she had finished and everybody answered with a number from 1 to 100. The sum of all answers equaled 4000. What is the smallest number of false answers the runners could have given?

A) 9 B) 10 C) 11 D) 12 E) 13

Q23) We throw a dice three times. If the number obtained on the third throw is equal to the sum of the numbers we obtained on the first two, what is the probability that a 2 appeared at least once?

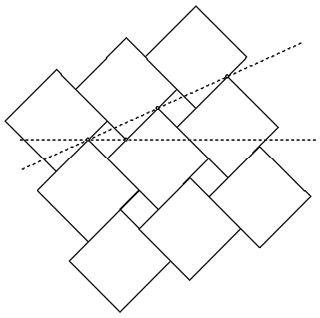
A) 1/6 B) 91/216 C) 1/2 D) 8/15 E) 7/12

Q24) A bar-code of the type shown is composed of alternate strips of black and white, always beginning and ending with a black strip. Each strip (of either colour) has the width 1 or 2, and the total width of the bar code is 12. How many different codes are possible, always reading from left to right?



A) 24 B) 132 C) 66 D) 12 E) 116

Q25) A wall is tiled with two sizes of square tile as shown. The larger tile has sides of length a, and the smaller of length b. The dashed lines (horizontal and slanted) form an angle of 30°. Determine the ratio a:b.



A) $(2\sqrt{3}) : 1$ B) $(2 + \sqrt{3}) : 1$ C) $(3 + \sqrt{2}) : 1$ D) $(3\sqrt{2}) : 1$ E) 2 : 1

Q26) The natural numbers from 1 to 10 are each written on the blackboard 10 times. The students in the class then play the following game: a student deletes 2 of the numbers and instead of them writes down on the blackboard their sum decreased by 1; after that another student deletes 2 of the numbers and instead of them writes down on the blackboard their sum decreased by 1; and so on. The game continues until only one number remains on the blackboard. The remaining number is:

A) less than 440 B) 451 C) 460 D) 488 E) more than 500

Q27) The value of the expression $\dfrac{(2+3)\left(2^2+3^2\right)\ldots\left(2^{1024}+3^{1024}\right)\left(2^{2048}+3^{2048}\right)+2^{4096}}{3^{2048}}$ equals:

A) 2^{2048} B) 2^{4096} C) 3^{2048} D) 3^{4096} E) $3^{2048} + 2^{2048}$

Q28) The square root $\sqrt{0.\underbrace{44\ldots4}_{100\ times}}$ is written as an infinite decimal. What is the 100^{th} digit after the decimal point?

A) 1 B) 2 C) 3 D) 4 E) 6

Q29) $f : R_+ \to R,\ \forall x > 0 \ :\ 2f(x) + 3f\left(\dfrac{2010}{x}\right) = 5x$, then

$f(6) = \ldots$

A) 993 B) 1 C) 2009 D) 1013 E) 923

Q30) Points *P* and *Q* are chosen, one on each leg of right-angled triangle. The length of the sides are *a* and *b* respectively. Let *K* and *H* be the feet of perpendiculars from *P* and *Q* respectively on the hypotenuse. Find the least possible value of the sum *KP+PQ+QH*.

A) $a + b$ B) $\dfrac{2ab}{a+b}$ C) $\dfrac{2ab}{\sqrt{a^2+b^2}}$ D) $\dfrac{(a+b)^2}{\sqrt{a^2+b^2}}$ E) $\dfrac{(a+b)^2}{2ab}$