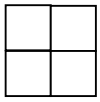
International Kangaroo Mathematics Contest 2008

Student Level: Class (11, 12 & 13) ***Max Time: 2 Hours***

3-point problems

1)

Numbers 3, 4 and two other unknown numbers are written in the cells of 2×2 table. It is known that the sums of the numbers in the rows are equal to 5 and 10, and the sum of the numbers in one of the columns is equal to 9. The larger of the two unknown numbers is



A) 5 B) 6 C) 7 D) 8 E) 3

2)

If $x + y = 0$ and $x \neq 0$, then $\frac{x^{2008}}{y^{2008}} =$

A) -1 B) 0 C) 1 D) 2^{2008} E) x/y

3)

An array contains 21 columns numbered 1, 2, … 21 and 33 rows numbered 1, 2, … 33. We erase the rows whose number is not a multiple of 3 and also the columns whose number is even. How many cells of the array remain after that?

A) 110 B) 121 C) 115,5 D) 119 E) 242

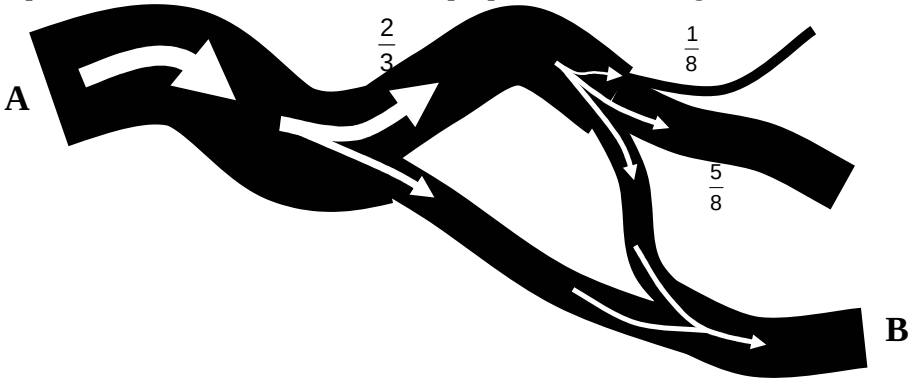
4)

How many prime numbers p have the property that $p^4 + 1$ is prime as well?

A) None B) 1 C) 2 D) 3 E) Infinitely many

5)

A river starts at point A. As it flows the river splits in two. The first branch takes 2/3 of the water and the second takes the rest. Later the first branch splits in three, one taking 1/8th of the branch's water, the second 5/8ths and the third the rest. Further down this last branch meets again a branch of the river. The map below shows the situation. What proportion of the original water flows at the point B?

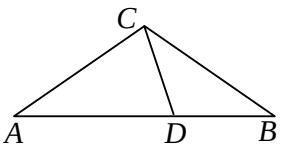


A) $\frac{1}{3}$ B) $\frac{5}{4}$ C) $\frac{2}{9}$ D) $\frac{1}{2}$ E) $\frac{1}{4}$

6)

Given an isosceles triangle ABC (CA = CB). The point D is marked on the side AB so that AD = AC and DB = DC (see the fig.). Find the value of the angle ACB.

A) 98° B) 100° C) 104°
D) 108° E) 110°



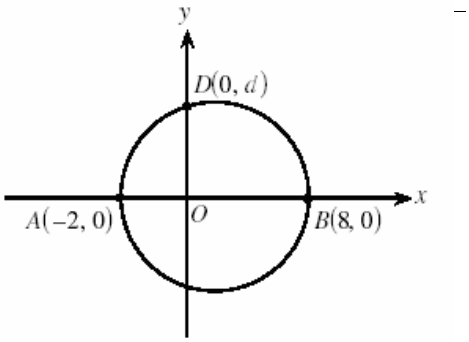
7)

The maximum value of $f(x) = |5\sin x - 3|$ for $x \in R$ is

A) 2 B) 3 C) π D) 5π E) 8

8)

The figure shows a circle with the diameter AB and point D on it. Find d.



A) 3 B) $2\sqrt{3}$ C) 4 D) 5 E) 6

9)

We have five different points A_1, A_2, A_3, A_4 and A_5, placed in this order on a straight line (with some distances between the points, that can be different). Another point P is placed on the same line so that the sum of the distances $PA_1 + PA_2 + PA_3 + PA_4 + PA_5$ is minimal. Then the point P is

A) A_1 B) A_2 C) A_3
D) Any point between A_2 and A_4 E) Any point between A_1 and A_5

10)

Shaheen wants to have on the empty places of 2 _ _ 8 two such digits that the complete number is divisible by 3. How many possibilities are there?

A) 29 B) 30 C) 19 D) 20 E) 33

4-point problems

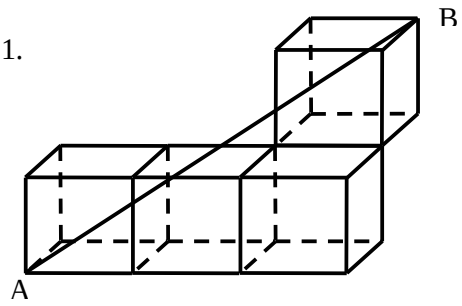
11)

Here are seven numbers: –9 ; 0 ; –5 ; 5 ; –4 ; –1 ; –3. We arranged six of them in groups of two so that the sum in each group is the same. Which number remains?

A) 5 B) 0 C) –3 D) –4 E) –5

12)

Each of the cubes in the figure has the length of edge equal to 1. What is the length of the segment AB?



A) $\sqrt{17}$ B) 7 C) $\sqrt{13}$
D) $\sqrt{7}$ E) $\sqrt{14}$

13)

Five problems are proposed on a Mathematical Competition. Since the problems have different difficulty level, no two of them have the same point value (all point values are positive integers). Nizami solved all five problems and he obtained a total of 10 points for the two problems with the lowest point value and a total of 18 points for the two problems with the highest point value. How many points did Bill obtain?

A) 30 B) 32 C) 34 D) 35 E) 40

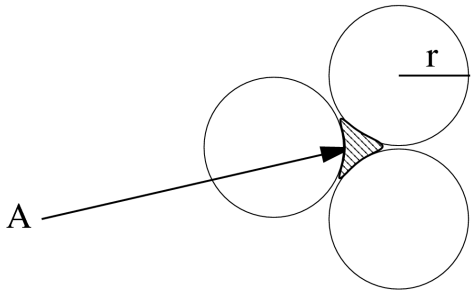
14)

Ayesha drew 36 kangaroos using three different colours. 25 of the kangaroos contain some yellow, 28 contain some brown and 20 contain some black colour. Only 5 of them have all the three colours. How many single-colour kangaroos did she draw?

A) None B) 4 C) 12 D) 31
E) It's impossible to know.

15)

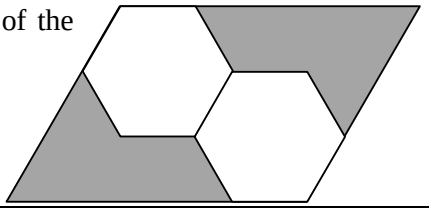
Three circles touch each other as shown. The radius of each circle is *r*. The area of *A* is



A) $\left(\sqrt{3}-\frac{1}{2}\pi\right)r^2$ B) $\left(\frac{1}{2}\pi-\frac{1}{2}\sqrt{3}\right)r^2$
C) $\frac{1}{8}\pi r^2$ D) $\left(\sqrt{3}-\frac{3}{2}\right)\pi r^2$ E) $\left(\frac{1}{3}\pi-\frac{1}{2}\sqrt{3}\right)r^2$

16)

In the figure the two regular hexagons are congruent. What fraction of the parallelogram's area is shaded?



A) 1/2 B) 1/3 C) 2/3
D) 2/5 E) 5/12

17)

The numerator and the denominator of a fraction are negative numbers, and the numerator is larger by one than the denominator. Which of the following is true about the fraction?

A) The fraction is a number less than -1.
B) The fraction is a number between -1 and 0.
C) The fraction is a positive number less than 1.
D) The fraction is a number greater than 1.
E) It cannot be determined whether the fraction is positive or negative.

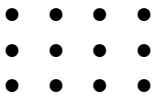
18)

Suppose $x^2yz^3 = 7^3$ and $xy^2 = 7^9$. Then $xyz =$

A) 7^4 B) 7^6 C) 7^8 D) 7^9 E) 7^{10}

19)

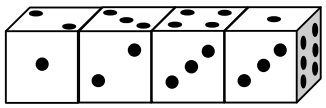
Three points are selected at random from the following grid. What is the probability that they are collinear?



A) $\frac{1}{12}$ B) $\frac{1}{11}$ C) $\frac{1}{16}$ D) $\frac{1}{8}$ E) $\frac{3}{12}$

20)

Four identical dice are arranged in a row (see the fig.). Each dice has faces with 1, 2, 3, 4, 5 and 6 points, but the dice are not standard, i. e. the sum of the points on the opposite faces of the dice does not necessarily equal 7. What is the total sum of the points in all 6 touching faces of the dice?



A) 19 B) 20 C) 21 D) 22 E) 23

5-point problems

21)

The lengths of the edges of a block (rectangular parallelepiped) in centimetres are integers and they form a geometric progression with quotient $q = 2$. Which of the following can be the volume of this solid?

A) 120 cm^3 B) 188 cm^3 C) 216 cm^3 D) 350 cm^3 E) 500 cm^3

22)

In the figure each asterisk stands for one digit. The sum of the digits of the product is equal to

$$\begin{array}{r} \times\ *\,*\,* \\ \quad 1\,*\,* \\ \hline \quad 2\,2\,*\,* \\ +\ \ 9\,0\,* \\ \quad *\,*\,2 \\ \hline 5\,6\,*\,*\,* \end{array}$$

A) 16 B) 20 C) 26 D) 30
E) Another answer

23)

Find the value of the expression $x^2 + y^2 + z^2$, if $x + y + z = 1$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$.

A) 0 B) 1 C) 2 D) 3
E) It is impossible to determine.

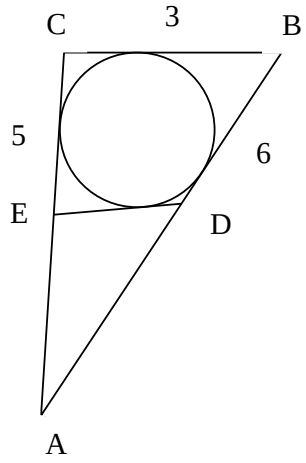
24)

The first element of a sequence is $a_1 = 0$, and if $n \geq 1$ then $a_{n+1} = a_n + (-1)^n \cdot n$. If $a_k = 2008$ then the value of k is

A) 2008 B) 2009 C) 4017 D) 4018 E) Other

25)

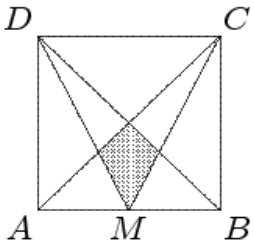
A circle is inscribed in the triangle ABC, as in the figure on the right, and |AC| = 5, |AB| = 6, |BC| = 3. The segment ED is tangent to the circle.
The perimeter of the triangle ADE is



A) 7 B) 4 C) 9
D) 6 E) 8

26)

The square ABCD has a side of length 1 and M is the midpoint of AB.
The area of the shaded region is



A) $\frac{1}{24}$ B) $\frac{1}{16}$ C) $\frac{1}{8}$ D) $\frac{1}{12}$ E) $\frac{2}{13}$

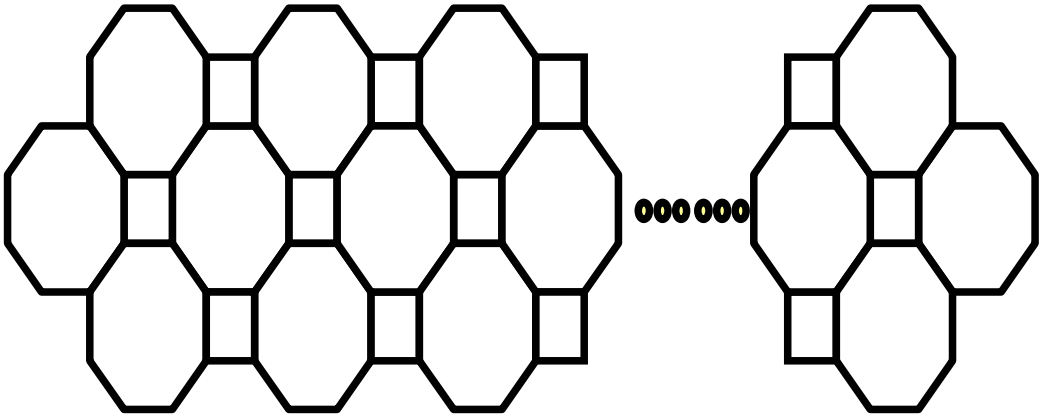
27)

A box contains seven cards. The cards are numbered from 1 to 7. Mary picks, at random, three cards from the box and afterwards John picks two cards. Two cards are left in the box. Then Mary says to John: "I know that the sum of the numbers of your cards is even." The sum of the numbers on Mary s cards is equal to

A) 10 B) 12 C) 6 D) 9 E) 15

28)

We used metal rods to build this nice ensemble. We know there are 61 octagons, how many rods are there?



A) 488 B) 400 C) 328 D) 244 E) 446

29)

The number $3^{32} - 1$ has exactly two divisors which are larger than 75 and smaller than 85. What is the product of these two divisors?

A) 5852 B) 6560 C) 6804 D) 6888 E) 6972

30)

If $\sin x + \cos x = m$, then $\sin^4 x + \cos^4 x =$

A) $1 - \frac{\left(1-m^2\right)^2}{2}$ B) $1 + \frac{\left(1-m^2\right)^2}{2}$ C) $\frac{1-\left(1-m^2\right)^2}{2}$ D) m^4 E) $m^4 + 1$

GOOD LUCK !