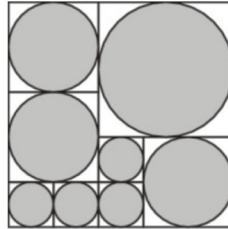


SECTION ONE - (3 point problems)

1. A large square is divided into smaller squares, as shown. A shaded circle is inscribed inside each of the smaller squares. What proportion of the area of the large square is shaded?



- (A) $\frac{8\pi}{9}$ (B) $\frac{13\pi}{16}$ (C) $\frac{3}{\pi}$
 (D) $\frac{3}{4}$ (E) $\frac{\pi}{4}$

2. How many integers are in the interval $(20 - \sqrt{21}, 20 + \sqrt{21})$?

- (A) 9 (B) 10 (C) 11
 (D) 12 (E) 13

3. A cube with edge 1 is cut into two identical cuboids. What is the surface area of one of these cuboids?

- (A) $\frac{3}{2}$ (B) 2 (C) 3
 (D) 4 (E) 5

4. Paula's weather app shows a diagram of the predicted weather and maximum temperatures for the next seven days, as shown. Which of the following represents the corresponding graph of maximum temperatures?

-1 °C	-4 °C	0 °C	0 °C	3 °C	-3 °C	-5 °C
Fri	Sat	Sun	Mon	Tue	Wed	Thu

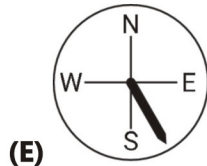
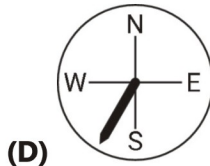
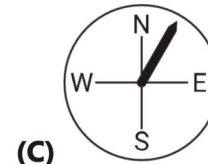
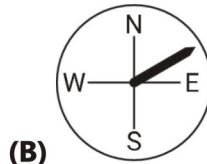
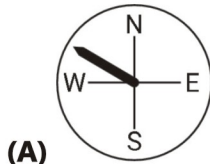
- (A)
- (B)
- (C)
- (D)
- (E)

31st INTERNATIONAL KANGAROO MATHEMATICS CONTEST 2021

KSF - Problems Student (Class 11 & 12)

Time Allowed: 180 minutes

5. After the storm last night, the flagpole on our school building is leaning over. Looking from northwest, its tip is to the right of its bottom point. Looking from the east, its tip is also to the right of its bottom point. In which direction could the flagpole be leaning over?



6. A rectangular sheet of paper has length x and width y , where $x > y$. The rectangle may be folded to form the curved surface of a circular cylinder in two different ways. What is the ratio of the volume of the longer cylinder to the volume of the shorter cylinder?

(A) $y^2 : x^2$

(B) $y : x$

(C) 1:1

(D) $x : y$

(E) $x^2 : y^2$

7. Let $x = \frac{\pi}{4}$. Which of the following numbers is the largest?

(A) x^4

(B) x^2

(C) x

(D) \sqrt{x}

(E) $\sqrt[4]{x}$

8. What is the area of the triangle whose vertices are at (p, q) , $(3p, q)$ and $(2p, 3q)$, where $p, q > 0$?

(A) $\frac{pq}{2}$

(B) pq

(C) $2pq$

(D) $3pq$

(E) $4pq$

9. How many 3-digit-numbers formed using only the digits 1, 3 and 5 are divisible by 3? You may use digits more than once.

(A) 3

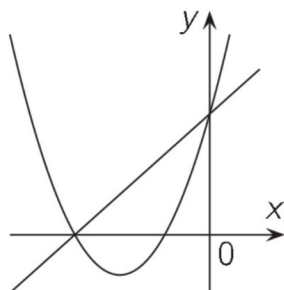
(B) 6

(C) 9

(D) 18

(E) 27

10. The parabola in the figure has an equation of the form $y = ax^2 + bx + c$ for some distinct real numbers a , b and c . Which of the following equations could be an equation of the line in the figure?



- (A) $y = bx + c$ (B) $y = cx + b$ (C) $y = ax + b$
 (D) $y = ax + c$ (E) $y = cx + a$

SECTION TWO - (4 point problems)

11. How many three-digit natural numbers have the property that when their digits are written in reverse order, the result is a three-digit number which is 99 more than the original number?

- (A) 8 (B) 64 (C) 72
 (D) 80 (E) 81

12. If $A = (0, 1) \cup (2, 3)$ and $B = (1, 2) \cup (3, 4)$, what is the set of all numbers of the form $a + b$ with a in A and b in B ?

- (A) $(1, 7)$ (B) $(1, 5) \cup (5, 7)$
 (C) $(1, 3) \cup (3, 7)$ (D) $(1, 3) \cup (3, 5) \cup (5, 7)$
 (E) none of the previous

13. What proportion of all the divisors of $7!$ is odd?

- (A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{4}$
 (D) $\frac{1}{5}$ (E) $\frac{1}{6}$

14. The first 1000 positive integers are written in a row in some order and all sums of any three adjacent numbers are calculated. What is the greatest number of odd sums that can be obtained?

- (A) 997 (B) 996 (C) 995
 (D) 994 (E) 993

31st INTERNATIONAL KANGAROO MATHEMATICS CONTEST 2021

KSF - Problems Student (Class 11 & 12)

Time Allowed: 180 minutes

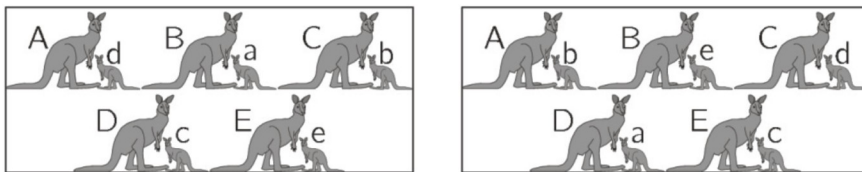
22. The numbers 1, 2, 7, 9, 10, 15 and 19 are written down on a blackboard. Two players alternately delete one number each until only one number remains on the blackboard. The sum of the numbers deleted by one of the players is twice the sum of the numbers deleted by the other player. What is the number that remains?

(A) 7 (B) 9 (C) 10
(D) 15 (E) 19

23. The function $f(x)$ is such that $f(x + y) = f(x) \cdot f(y)$ and $f(1) = 2$. What is the value of $\frac{f(2)}{f(1)} + \frac{f(3)}{f(2)} + \dots + \frac{f(2021)}{f(2020)}$??

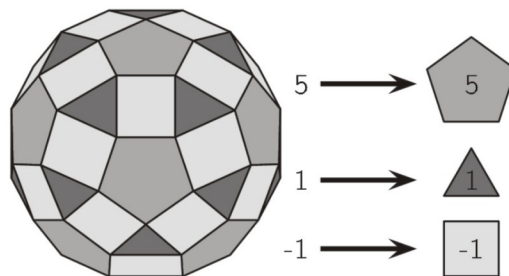
(A) 0 (B) $\frac{1}{2}$ (C) 2
(D) 2020 (E) none of the previous

24. Five kangaroos named A, B, C, D and E have one child each, named a, b, c, d and e. In the first group photo shown exactly two of the children are standing next to their mothers. In the second group photo exactly three of the children are standing next to their mothers. Whose child is a?



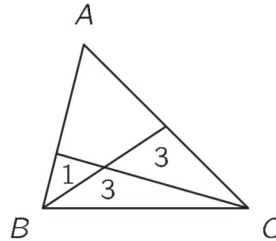
(A) A (B) B (C) C
(D) D (E) E

25. The solid shown in the diagram has 12 regular pentagonal faces, the other faces being either equilateral triangles or squares. Each pentagonal face is surrounded by 5 square faces and each triangular face is surrounded by 3 square faces. John writes 1 on each triangular face, 5 on each pentagonal face and -1 on each square. What is the total of the numbers written on the solid?

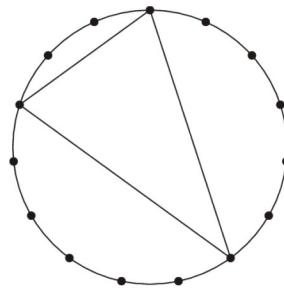


(A) 20 (B) 50 (C) 60
(D) 80 (E) 120

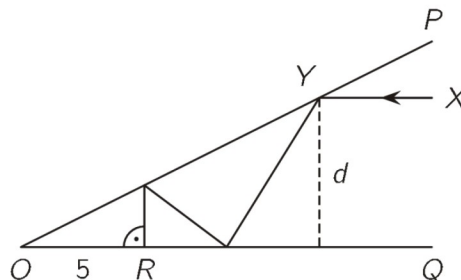
26. A triangle ABC is divided into four parts by two straight lines, as shown. The areas of the smaller triangles are 1, 3 and 3. What is the area of the original triangle?



- (A) 12
(B) 12.5
(C) 13
(D) 13.5
(E) 14
27. On a circle 15 points are equally spaced. We can form triangles by joining any three of these. We count two triangles as being the same if they are congruent i.e. one is a rotation and/or a reflection of the other. How many different triangles can be drawn?



- (A) 19
(B) 91
(C) 46
(D) 455
(E) 23
28. Two plane mirrors OP and OQ are inclined at an acute angle (diagram is not to scale). A ray of light XY parallel to QO strikes mirror OP at Y . The ray is reflected and hits mirror OQ , is reflected again and hits mirror OP and is reflected for a third time and strikes mirror OQ at right angles at R , as shown. The distance OR is 5 cm. The ray XY is d cm from the mirror OQ . What is the value of d ?



- (A) 4
(B) 4.5
(C) 5
(D) 5.5
(E) 6

31st INTERNATIONAL KANGAROO MATHEMATICS CONTEST 2021

KSF - Problems Student (Class 11 & 12)

Time Allowed: 180 minutes

29. A certain game is won when one player gets 3 points ahead. Two players A and B are playing the game and at a particular point, A is 1 point ahead. Each player has an equal probability of winning each point. What is the probability that A wins the game?

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) $\frac{3}{4}$

(D) $\frac{4}{5}$

(E) $\frac{5}{6}$

30. Let $M(k)$ be the maximum value of $|4x^2 - 4x + k|$ for x in the interval $[-1,1]$, where k can be any real number. What is the minimum possible value of $M(k)$?

(A) 4

(B) $\frac{9}{2}$

(C) 5

(D) $\frac{11}{2}$

(E) 8

-- Good Luck --