3 point problems

1. Andrea was born in 1997, her younger sister Charlotte in 2001. The age difference of the two sisters is therefore in any case.

(**A**) less than 4 years (**B**) at least 4 years (**C**) exactly 4 years
(**D**) more than 4 years (**E**) not less than 3 years

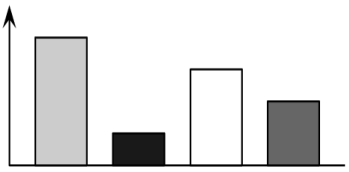
2. $(a-b)^5 + (b-a)^5 =$

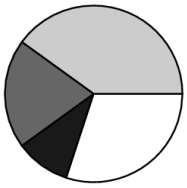
(**A**) 0 (**B**) $2\,(a-b)^5$ (**C**) $2a^5 - 2b^5$ (**D**) $2a^5 + 2b^5$
(**E**) $2a^5 + 10a^4b + 20a^3b^2 + 20a^2b^3 + 10ab^4 + 2b^5$

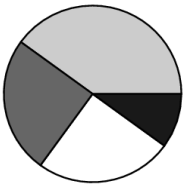
3. How many solutions does the equation $2^{2x} = 4^{x+1}$ have?

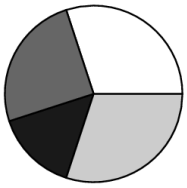
(**A**) 0 (**B**) Infinitely many (**C**) 2 (**D**) 1 (**E**) 3

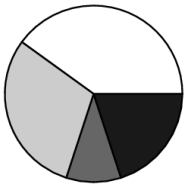
4. Diana drew a bar chart representing the quantity of the four tree species registered during a biology excursion. Jasper thinks that a circular chart would better represent the ratios of the different tree species. What does the respective circular chart look like?

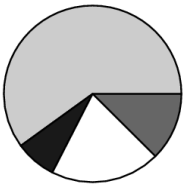


(**A**) 

(**B**) 

(**C**) 

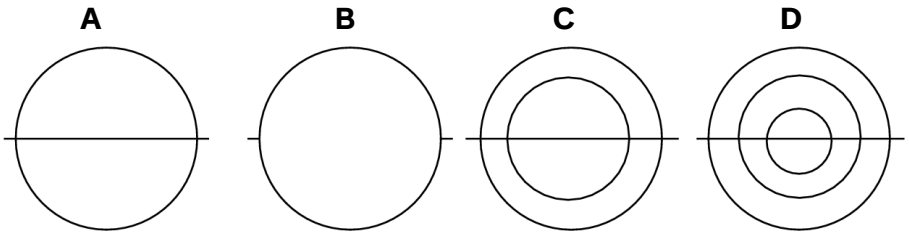
(**D**) 

(**E**) 

5. We add the 31 integers from 2001 to 2031 and divide the sum by 31. What result do we get?

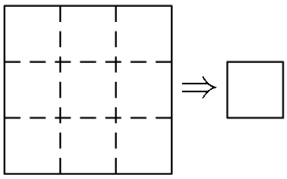
(**A**) 2012 (**B**) 2013 (**C**) 2015 (**D**) 2016 (**E**) 2496

6. How many of the following figures can be drawn with one continuous line without drawing a segment twice?



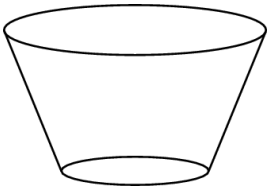
(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

7. A square piece of paper is folded along the dashed lines one after the other in any order or direction. From the resulting square one corner is cut off. Now the paper is unfolded. How many holes are in the paper?

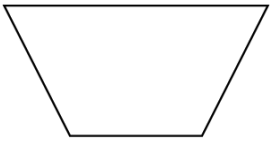


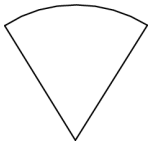
(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 4 (**E**) 9

8. A drinking glass has the shape of a truncated cone (see figure). The outside of the glass (without the base) should now be covered with colored paper. What shape does the paper need to be in order to completely cover the whole glass without overlaps?

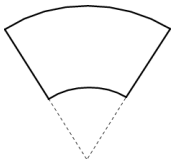


(**A**) rectangle 

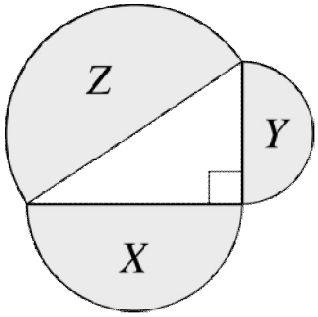
(**B**) trapezoid 

(**C**) circle sector 

(**D**) parallel strip 

(**E**) part of a circle sector 

9. Three semicircles have diameters which are the sides of a right-angle triangle. Their areas are X cm², Y cm² and Z cm², as shown. Which of the following is necessarily true?



(**A**) $X + Y < Z$ (**B**) $\sqrt{X} + \sqrt{Y} = \sqrt{Z}$ (**C**) $X + Y = Z$
(**D**) $X^2 + Y^2 = Z^2$ (**E**) $X^2 + Y^2 = Z$

10. Which of the following is the complete list of the number of acute angles a convex quadrilateral can have?

(**A**) 0, 1, 2 (**B**) 0, 1, 2, 3 (**C**) 0, 1, 2, 3, 4
(**D**) 0, 1, 3 (**E**) 1, 2, 3

4 point problems

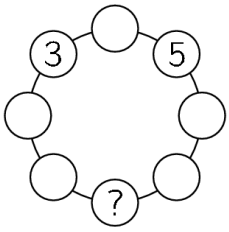
11. $\sqrt{(2015 + 2015) + (2015 - 2015) + (2015 \cdot 2015) + (2015 : 2015)}$ =

(**A**) $\sqrt{2015}$ (**B**) 2015 (**C**) 2016 (**D**) 2017 (**E**) 4030

12. The x-axis and the graphs of the functions $f(x) = 2 - x^2$ and $g(x) = x^2 - 1$ split the Cartesian plane into

(**A**) 7 regions (**B**) 8 regions (**C**) 9 regions
(**D**) 10 regions (**E**) 11 regions

13. Ella wants to write a number in each circle in the picture such that each number is the sum of its two neighbours. Which number must Ella write in the circle with the question mark?



(**A**) -5 (**B**) -16 (**C**) -8 (**D**) -3 (**E**) This is impossible.

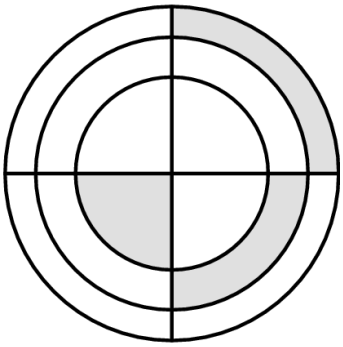
14. Given five different positive integers a, b, c, d, e, we know that $c : e = b$, $a + b = d$ and $e - d = a$. Which of the numbers a, b, c, d, e is the largest?

(**A**) a (**B**) b (**C**) c (**D**) d (**E**) e

15. The geometric mean of a set of n positive numbers is defined as the n-th root of the product of those numbers. The geometric mean of a set of three numbers is 3 and the geometric mean of another set of three numbers is 12. What is the geometric mean of the combined set of six numbers?

(**A**) 4 (**B**) 6 (**C**) $\frac{15}{2}$ (**D**) $\frac{15}{6}$ (**E**) 36

16. In the figure shown there are three concentric circles and two perpendicular diameters. If the three shaded figures have equal area and the radius of the small circle is one, what is the product of the three radii?



(**A**) $\sqrt{6}$ (**B**) 3 (**C**) $\frac{3\sqrt{3}}{2}$ (**D**) $2\sqrt{2}$ (**E**) 6

17. An automobile dealer bought two cars. He sold the first one for 40% more than he paid for it and the second one for 60% more than he paid for it. The money he received for the two cars was 54% more than what he paid for both. The ratio of the prices the dealer paid for the first and the second car was:

(**A**) 10:13 (**B**) 20:27 (**C**) 3:7 (**D**) 7:12 (**E**) 2:3

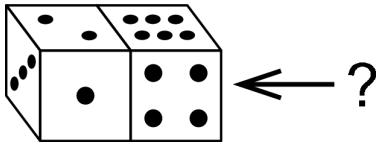
18. Bibi has a die with the numbers 1, 2, 3, 4, 5 and 6 on its six faces. Tina has a die which is special: it has the numbers 2, 2, 2, 5, 5 and 5 on its six faces. When Bibi and Tina roll their dice the one with the larger number wins. If the two numbers are equal it is a draw. What is the probability that Tina wins?

(**A**) $\frac{1}{3}$ (**B**) $\frac{7}{18}$ (**C**) $\frac{5}{12}$ (**D**) $\frac{1}{2}$ (**E**) $\frac{11}{18}$

19. There are 2015 marbles in a cane. The marbles are numbered from 1 to 2015. Marbles with equal digit sums have the same color and marbles with different digit sums have different colors. How many different colors of marbles are there in the cane?

(**A**) 10 (**B**) 27 (**C**) 28 (**D**) 29 (**E**) 2015

20. For standard dice the sum of the numbers on opposite faces is 7. There are two identical standard dice shown in the figure. What number may be on the (not visible) face on the right (marked by the "?" sign)?



(**A**) Only 5 (**B**) Only 2 (**C**) Either 2 or 5
(**D**) Either 1, 2, 3 or 5 (**E**) Either 2, 3 or 5

5 point problems

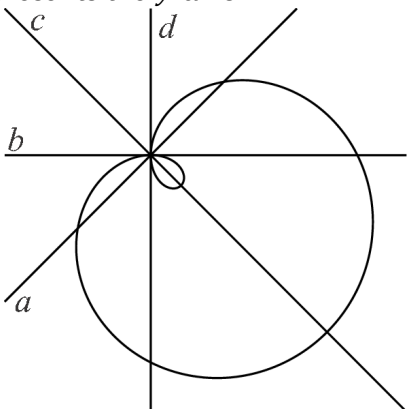
21. The following is the multiplication table of the numbers 1 to 10.

x	1	2	3	...	10
1	1	2	3	...	10
2	2	4	6	...	20
: .	: .				: .
10	10	20	30	...	100

What is the sum of all 100 products in the complete table?

(**A**) 1000 (**B**) 2025 (**C**) 2500 (**D**) 3025 (**E**) 5500

22. The curve in the figure is described by the equation $(x^2 + y^2 - 2x)^2 = 2(x^2 + y^2)$
Which of the lines a, b, c, d represents the y-axis?



(**A**) a (**B**) b (**C**) c (**D**) d (**E**) none of these

23. When reading the following statements from the left to the right what is the first statement which is true?

(**A**) (C) is true. (**B**) (A) is true. (**C**) (E) is false. (**D**) (B) is false. (**E**) 1 + 1 = 2.

24. How many regular polygons exist such that their angles (in degrees) are integers?

(**A**) 17 (**B**) 18 (**C**) 22 (**D**) 25 (**E**) 60

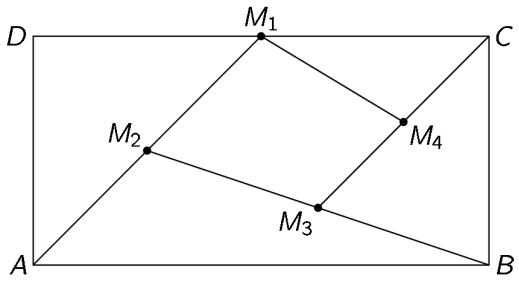
25. How many 3-digit positive integers can be represented as the sum of exactly nine different powers of 2?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

26. How many triangles ABC with $\angle ABC = 90°$ and $AB = 20$ exist such that all sides have integer lengths?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 6

27. In the rectangle $ABCD$ shown in the figure, M_1 is the midpoint of DC, M_2 is the midpoint of AM_1, M_3 is the midpoint of BM_2 and M_4 is the midpoint of CM_3. Find the ratio between the areas of the quadrilateral $M_1M_2M_3M_4$ and of the rectangle $ABCD$.



(**A**) $\frac{7}{16}$ (**B**) $\frac{3}{16}$ (**C**) $\frac{7}{32}$ (**D**) $\frac{9}{32}$ (**E**) $\frac{1}{5}$

28. Blue and red rectangles are drawn on a blackboard. Exactly 7 of the rectangles are squares. There are 3 red rectangles more than blue squares. There are 2 red squares more than blue rectangles. How many blue rectangles are there on the blackboard?

(**A**) 1 (**B**) 3 (**C**) 5 (**D**) 6 (**E**) 10

29. 96 members of a counting club are standing in a large circle. They start saying numbers 1, 2, 3, etc. in turn, going around the circle. Every member that says an even number steps out of the circle and the rest continue, starting the second round with 97. They continue in this way until only one member is left. Which number did this member say in the first round?

(**A**) 1 (**B**) 17 (**C**) 33 (**D**) 65 (**E**) 95

30. In the word KANGAROO Bill and Bob replace the letters by digits, so that the resulting numbers are multiples of 11. They each replace different letters by different digits and the same letters by the same digits ($K \neq 0$). Bill obtains the largest possible such number and Bob the smallest. In both cases one of the letters is replaced by the same digit. Which digit is this?

(**A**) 0 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6